THE CALCULATION AND MEASUREMENT OF INDUCTANCE AND CAPACITY

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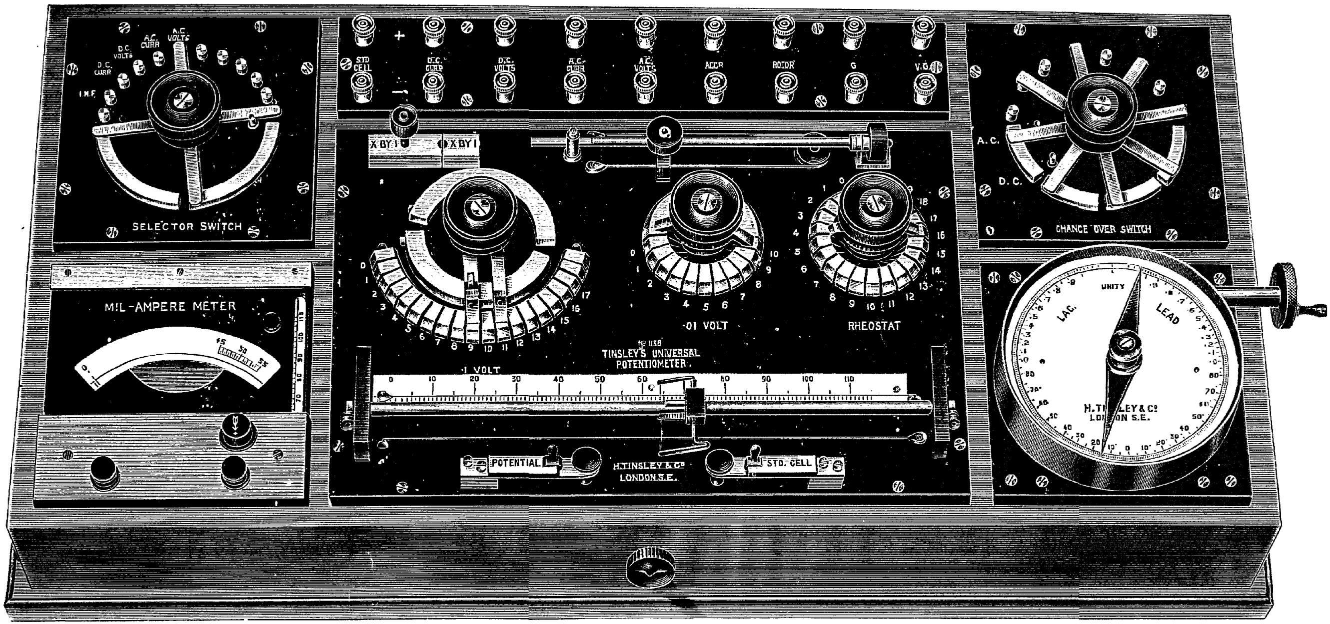

Dr. C. V. Drysdale's Alternating-Current Potentiometer. [*Frontispiece.*

(*Page* 68.)

THE CALCULATION AND MEASUREMENT OF INDUCTANCE AND CAPACITY

BY

W. H. NOTTAGE, B.Sc.

FELLOW OF THE PHYSICAL SOCIETY OF LONDON, ASSOCIATE OF THE INSTITUTE OF RADIO ENGINEERS, LATE LECTURER IN WIRELESS TELEGRAPHY AT THE BATTERSEA POLYTECHNIC

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PREFACE

The object of this book is to bring together, in a convenient form, the more generally useful formulas and methods of measurement for inductance and capacity.

There have been considerable developments in recent years in the design of instruments intended for these measurements, and it is hoped that the descriptions herein may prove of service to those engaged in the work, especially as some do not appear to have been yet noticed in the various text-books dealing with the subject.

Every endeavour has been made to make the collection accurate, but the author will be obliged by a note of any errors which may have escaped notice, or of suggestions as to better methods or formulas.

The author desires to place on record his obligations to Professor G. W. O. Howe, for permission to make extensive use of his valuable articles on the capacity of radiotelegraphic antennæ; to the Cambridge Scientific Instrument Co., Messrs. Muirhead and Co., Mr. R. W. Paul, and Messrs. H. Tinsley and Co., for information concerning the instruments manufactured by them, and for the loan of blocks for illustrations, and to Mr. C. S. Agate, B.Sc., for assistance in reading the proofs.

W. H. N.

Marconi Works, Chelmsford,
October, 1916.

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INDUCTANCE AND CAPACITY

CHAPTER I

THE CALCULATION OF INDUCTANCE

When an electric conductor is placed in a magnetic field, and by some means the field within the conducting circuit varies, either by their relative motion or by the varying in strength of the field, then an electromotive force will be created in the conductor, and if it form a closed metallic circuit a current will flow in it.

Every electric current has associated with it a magnetic field, the strength of which at any point in the neighbourhood depends on the geometrical configuration of the circuit carrying the current, and also on the magnitude of the current. Hence when the current varies the associated magnetic field varies, and an electromotive force will be generated in any conductor placed in the varying field. An electromotive force will also be generated in the circuit carrying the current, since this itself is in the varying field.

The relationship between the E.M.F. induced and the current to which the field is due is given by

$$E = -M\frac{dc}{dt}$$

when the E.M.F. is induced in an independent circuit, and by

$$E = -L\frac{dc}{dt}$$

when it is induced in the same circuit as the current.

M is a quantity termed the coefficient of mutual inductance, or, more shortly, the mutual inductance between the circuits; and similarly L is the self-inductance of the circuit carrying the current.

In the case of all circuits in which the magnetic field produced is directly proportional to the current, the mutual and self-inductance may be calculated from their geometrical dimensions, but for circuits which have iron or other magnetic substance associated

with them in the form of a core, for which the magnetic field produced is not directly proportional to the current strength, the properties of this core must be taken into account.

The absolute unit of inductance in the electromagnetic system is usually termed the "centimetre."

Inductances worked out by the formulas in the following pages are given in these units, except for certain formulas where the inductances are in henrys as stated.

The henry is the practical unit of inductance which corresponds to the ohm, ampere, and volt.

The inductances measured by the methods in Chapter II. are for the greater part expressed in henrys, since the resistances are reckoned in ohms.

The millihenry and microhenry are convenient units for expressing the values of inductances in many cases.

The relationship between these units is

1 henry $= 10^9$ electromagnetic units (centimetres).
1 millihenry $= 10^3$ microhenrys $= 10^6$ centimetres.
1 microhenry $= 10^3$ centimetres.

Inductance of a Straight Cylindrical Wire.

I. *When carrying continuous current.*

The inductance is given by the following formula, due to Neumann :—

$$L = 2l\left(\log_e \frac{2l}{r} - 1 + \frac{\mu}{4}\right) \quad . \quad . \quad . \quad . \quad (1)$$

where l = length of wire in centimetres,
r = radius " "
μ = permeability of the wire.

This formula assumes that the medium outside the wire is of unit permeability (*i.e.* that it is not, for example, in a slot in an iron circuit), and also that the radius is small compared with the length.

For most wires, except of iron and similar materials, the permeability will be unity also, so that the formula becomes

$$\mathbf{L} = 2l\left(\log_e \frac{2l}{r} - \frac{3}{4}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

II. *When carrying low-frequency alternating current.*

The inductance is given by

$$L = L_0 - \frac{\mu l x^2}{48}\left(1 - \frac{13}{180}x^2 + \ldots\right) \quad . \quad . \quad . \quad (3)$$

where

$$x = \frac{\mu p a}{\rho},$$

L_0 = inductance for continuous current found by formula (1) or (2),
a = sectional area of the wire in square centimetres,
ρ = specific resistance in C.G.S. units,
$p = 2\pi n$, where n is the frequency.

The above formula may be used for values of x below 2; for higher values:

$$L = L_0 - \frac{\mu l}{2}\left(1 - \sqrt{\frac{2}{x}}\right) \quad . \quad . \quad . \quad . \quad (4)$$

(Heaviside and also Rayleigh.)

III. *When carrying alternating current of high frequency.*

For a wire carrying current of which the frequency is high, such as used for wireless telegraphy, then, since the current is confined to the surface layers of the wire, so that there is no magnetic field within it, the most convenient formula is

$$\mathbf{L} = 2l\left(\log_e \frac{2l}{r} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or

$$L = L_0 - \frac{\mu l}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where L_0 is the inductance for continuous current.

These formulas give the continuous-current inductance of a thin tube.

Example.—Calculate the inductance of a length of 1 metre of No. 10 S.W.G. copper wire for continuous current, high frequency, and A.C. of 1000 periods.

Diameter of No. 10 S.W.G. wire = ·325 cm.

Therefore $$\frac{2l}{r} = \frac{200}{\cdot 325} = 615{\cdot}39$$

$$^{*} \log_e 615{\cdot}39 = 6{\cdot}4222$$

By formula (2) the continuous-current inductance is

$$200(6{\cdot}4222 - \tfrac{3}{4}) = 1134{\cdot}4 \text{ centimetres}$$

or 1·134 microhenrys

By formula (5) the high-frequency inductance is

$$200(6{\cdot}4222 - 1) = 1084{\cdot}4 \text{ cms.}$$

= 1·084 microhenrys

* The logarithm to base e of a number n is given by

$$\log_e n = \log_{10} n \times 2{\cdot}3026$$

For the inductance at 1000 cycles we have

$$x = \frac{\mu p a}{\rho}$$
$$\mu = 1$$
$$p = 2\pi 1000$$
$$a = \cdot 08302 \text{ sq. cms.}$$
$$\rho = 1700 \text{ C.G.S. for copper (at } 60^\circ \text{ F.)}$$

Hence
$$x = \frac{2\pi 1000 \times \cdot 08302}{1700} = \cdot 3068$$
$$x^2 = \cdot 09414$$

Therefore the inductance =

$$\mathrm{L} = \mathrm{L}_0 - \frac{\mu l x^2}{48}\left(1 - \frac{13}{180}x^2\right) \text{ by formula (3)}$$
$$= 1134{\cdot}4 - \frac{200 \times \cdot 09414}{48}\left(1 - \frac{13}{180} \times \cdot 09414\right)$$
$$= 1134{\cdot}4 - \cdot 3922(1 - \cdot 0068)$$
$$= 1134{\cdot}4 - \cdot 3725$$
$$= 1134 \text{ cms.}$$
$$= 1{\cdot}134 \textbf{ microhenrys}$$

The inductance is therefore only slightly different from that for continuous current.

The above formulas for alternating current inductance assume that the wave form of the current is undamped. For damped waves a correction is necessary in cases where great accuracy is required.

For the inductance at high frequencies the formula is

$$\mathrm{L} = \mathrm{L}_0 - \mu l\left(\frac{1}{2} - \sqrt{\frac{s(s+k)}{2x}}\right) \quad . \quad . \quad . \quad (7)$$

(Barton.)

where L_0 is the continuous-current inductance as before;

or
$$\mathrm{L} = \mathrm{L}_1 + \frac{\mu l}{\sqrt{2x}}(\sqrt{s(s+k} - 1) \quad . \quad . \quad . \quad (7a)$$

where L_1 is the inductance found by formula (5).

x has the same value as in formula (3), page 8.

$$k = \frac{\delta}{2\pi} \text{ and } s = \sqrt{1 + k^2}$$

where δ is the decrement of the vibration per complete period.

To see the magnitude of this correction, apply it to the example worked out above, taking the frequency at 10^6 and $k = \cdot 04$.

$$s = 1{\cdot}0008$$
$$s(s+k) = 1{\cdot}0416$$
$$x = 306{\cdot}8$$

Hence
$$\begin{aligned} L &= L_0 - l\left(\frac{1}{2} - \sqrt{\frac{1{\cdot}0416}{613{\cdot}6}}\right) \\ &= L_0 - l\,({\cdot}5 - {\cdot}0414) \\ &= L_0 - 100({\cdot}4586) \\ &= 1134{\cdot}4 - 46{\cdot}0 \\ &= 1088{\cdot}4 \text{ cms.} \end{aligned}$$

which is very little different from the value for undamped waves.

Thin Tape.

For many purposes the conductors, instead of being cylindrical wires, are thin tapes.

The inductance of a thin tape is given by

$$\mathbf{L} = 2l\left(\log_e \frac{2l}{b} + \frac{1}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

where the thickness of the tape is negligible.

Where it is not negligible the formula becomes

$$L = 2l\left(\log_e \frac{2l}{t+b} + \frac{1}{2} + \frac{{\cdot}2235(t+b)}{l}\right) \quad . \quad . \qquad (8a)$$

(Maxwell.)

where t = thickness,
and b = breadth of tape in cms.

This formula, therefore, applies to a conductor of rectangular cross-section.

Example.—Calculate the inductance of 5 metres of thin tape 2 cms. broad, ·5 mm. thick.

By formula (8*a*)

$$\begin{aligned} L &= 2 \times 500\left(\log_e \frac{2 \times 500}{2{\cdot}05} + \frac{1}{2} + \frac{{\cdot}2235(2{\cdot}05)}{500}\right) \\ &= 1000\left(\log_e 497{\cdot}6 + {\cdot}5 + \frac{{\cdot}4582}{500}\right) \\ &= 1000(6{\cdot}2099 + {\cdot}5 + {\cdot}0009) \\ &= 1000 \times 6{\cdot}7108 \text{ cms.} \\ &= 6{\cdot}71 \text{ microhenrys} \end{aligned}$$

By the approximate formula No. 8

$$\begin{aligned} L &= 1000(\log_e 500 + {\cdot}5) \\ &= 1000(6{\cdot}2146 + {\cdot}5) \\ &= 6{\cdot}715 \text{ microhenrys} \end{aligned}$$

Calculate the inductance if the tape be 2·5 mm. thick.

$$L = 1000\left(\log_e \frac{1000}{2{\cdot}25} + \frac{1}{2} + \frac{{\cdot}2235 \times 2{\cdot}25}{500}\right)$$
$$= 1000(\log_e 444{\cdot}4 + {\cdot}5 + 0010)$$
$$= 6{\cdot}598 \text{ microhenrys}$$

Return Circuit of Two Parallel Wires or Tapes.

For a circuit in whch the current flows in by one wire and returns by a similar, parallel, one, the total inductance of the circuit is given by

$$L = L_1 + L_2 - 2M$$

where L_1 and L_2 are the self-inductances of the two wires, and M is the mutual inductance between them.

M can be calculated from the formulas on page 3, but for the case of similar wires of equal length the total inductance is given by the following formulas :—

Cylindrical wires.

For continuous current

$$L = 4l\left(\log_e \frac{d}{r} + \frac{\mu}{4} - \frac{d}{l}\right) \quad . \quad . \quad . \quad . \quad (9)$$

neglecting any effect due to the cross-connection at the end,

where d = distance apart,
l = length of one wire,
r = radius " "
all measured in cms.

For the case where $\mu = 1$ and $\frac{d}{l}$ is small, the above reduces to

$$\mathbf{L} = \mathbf{4}l\left(\mathbf{\log_e} \frac{\mathbf{d}}{\mathbf{r}} + \frac{\mathbf{1}}{\mathbf{4}}\right). \quad . \quad . \quad . \quad . \quad (10)$$

When the current is not continuous, the inductance is given by

$$L = 4l\left(\log_e \frac{d}{r} - \frac{d}{l} + y\right) \quad . \quad . \quad . \quad . \quad (11)$$

where the value of y for

low-frequency currents is $\frac{\mu}{4} - \frac{\mu x^2}{96}\left(1 - \frac{13}{180}x^3 \; . \; . \; .\right)$

high-frequency " $\frac{\mu}{4}\sqrt{\frac{2}{x}}$

very high " " 0

where $x = \frac{\mu p a}{\rho}$ as before (page 9).

The inductance for high-frequency currents is therefore given by

$$L = 4l\left(\log_e \frac{d}{r}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Parallel tapes.

The inductance of a return circuit formed by two parallel tapes is given by

$$\begin{aligned} L &= 2L_1 - 2M \\ &= 4l \log_e \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13) \end{aligned}$$

where R_2 is a quantity termed the geometrical mean distance of one tape from the other, and R_1 is the geometrical mean distance of one tape from itself.

If the tapes be spaced apart a distance equal to their width, the formula simplifies to

$$L = 2\pi l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and the inductance is therefore independent of the width, provided the spacing is equal to this width.

The above formulas (13) and (14) are correct for all frequencies and continuous current, the thickness of the tape being neglected.

Geometric Mean Distance.

The geometric mean distance is a quantity which enters into several formulas for calculating inductance. It is often referred to as G.M.D. for brevity.

The geometric mean distance R of a point from a line is given by the nth root of the product of the n distances from the point to the line n being increased to infinity in determining R.

The point may be on the line itself, and by taking all possible points on the line we can obtain the G.M.D. of the line from itself, and similarly the G.M.D. of areas can be calculated.

The following are some values for the G.M.D. of various figures :—

Straight line.

The G.M.D. of a line of length a from itself is given by

$$\log_e R = \log_e a - \tfrac{3}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

or

$$R = \cdot 22313a \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Rectangular area.

An approximate expression for the G.M.D. is

$$R = \cdot 2235(a + b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where a and b are the sides of the rectangle.

Square area.

$$R = \cdot 44705a \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Circular area.

$$\log R = \log a - \tfrac{1}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

or

$$R = \cdot 7788a$$

a being the radius of the circle.

Two straight lines.

I. When the lines are of equal length and are in the same straight line

$$\begin{aligned} R_0 &= \ \cdot 2231a \\ R_1 &= \ \cdot 8925a \\ R_2 &= 1\cdot 9565a \\ R_3 &= 2\cdot 9717a \\ R_4 &= 3\cdot 9789a \\ R_5 &= 4\cdot 9832a \\ R_6 &= 5\cdot 9861a \\ R_7 &= 6\cdot 9881a \\ R_8 &= 7\cdot 9896a \\ R_9 &= 8\cdot 9908a \quad . \quad . \quad . \quad . \quad . \quad . \quad (20) \end{aligned}$$

where R_0 is for the case where the lines are in contact,

R_1 for when they are separated by a distance equal to their length,

R_2 is for when they are separated by a distance equal to twice their length, and so on.

II. When the lines are parallel and distant d from each other, and of length b

$$\log_e R = \frac{d^2}{b^2}\log d + \frac{1}{2}\left(1 - \frac{d^2}{b^2}\right)\log (b^2 + d^2)$$
$$+ 2\frac{d}{b}\tan^{-1}\frac{b}{d} - \frac{3}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

when $d = b$

$$\log R = \log b + \frac{\pi}{2} - \frac{3}{2} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Circle from a point outside.

The G.M.D. of a point P outside a circle is equal to the distance from P to the centre of the circle . . . . (23)

Example.—Calculate the inductance of a return circuit of 5 metres, flat tape 3·5 cms. wide, 1·5 cm. spacing between the tapes.

By formula (13)

$$L = 4l\log_e\frac{R_2}{R_1} = 4l(\log_e R_2 - \log_e R_1)$$

By formula (21) *

$$\log_e R_2 = \frac{d^2}{b^2}\log d + \frac{1}{2}\left(1 - \frac{d^2}{b^2}\right)\log_e (b^2 + d^2) + 2\frac{d}{b}\tan^{-1}\frac{b}{d} - \frac{3}{2}$$

$$\frac{d}{b} = \frac{1{\cdot}5}{3{\cdot}5}; \quad \frac{d^2}{b^2} = {\cdot}1837$$

$$\begin{aligned}
\log_e R_2 &= {\cdot}1837 \log_e 1{\cdot}5 + \tfrac{1}{2}({\cdot}8163)\log_e (14{\cdot}5) + 2 \times \tfrac{3}{7}\tan^{-1}\tfrac{7}{3} - \tfrac{3}{2} \\
&= {\cdot}1837 \times {\cdot}4055 + {\cdot}4081 \times 2{\cdot}6742 + {\cdot}8556 \times 1{\cdot}166 - 1{\cdot}5 \\
&= {\cdot}7449 + 1{\cdot}0910 + {\cdot}9977 - 1{\cdot}5 \\
&= 1{\cdot}336
\end{aligned}$$

$$\log_e R_1 = \log_e 3{\cdot}5 - 1{\cdot}5 \qquad \text{by formula (15)}$$

$$\begin{aligned}
\text{Hence } L &= 2000(1{\cdot}336 - 1{\cdot}2528 + 1{\cdot}5) \\
&= 2000(1{\cdot}583) \\
&= 3{\cdot}166 \text{ microhenrys}
\end{aligned}$$

Inductance of a Rectangle.

I. *Cylindrical wire.*

The inductance of a rectangle of round wire is given by

$$L = 4\left\{(a+b)\log_e \frac{2ab}{r} - a\log_e (a+d) - b\log_e (b+d) - y(a+b) + 2(d+r)\right\} \quad . \quad . \quad . \quad . \quad (24)$$

where a and b are the sides,
and $d = \sqrt{a^2 + b^2}$ is the diagonal of the rectangle,
r = the radius of the wire used,
y = the factor given by

For continuous current

$$y = 2 - \frac{\mu}{4} = 1{\cdot}75 \text{ for ordinary cases}$$

for low frequency $\quad y = 2 - \dfrac{\mu}{4} + \dfrac{\mu x^2}{96}\left(1 - \dfrac{13}{180}x^2\right)$

for high frequency $\quad y = 2 - \dfrac{\mu}{4}\sqrt{\dfrac{2}{x}}$

for very high frequency

$$y = 2$$

$$x \text{ is } \frac{\mu p \pi r^2}{\rho} \text{ as before}$$

The above formulas should be used for the case of parallel wires when the ends cannot be neglected.

* b is here the *breadth* of the tape.

Square of cylindrical wire.

When $a = b$ the circuit becomes a square, and the formula simplifies to

$$L = 8a\left(\log_e \frac{a}{r} + \frac{r}{a} + 1{\cdot}226 - y\right) \quad . \quad . \quad . \quad (25)$$

Circle.

The inductance of a circle of which the cross-section is small compared with the diameter of the circle is given by Maxwell's formula

$$L = 4\pi a\left\{\left(1 + \frac{3R^2}{16a^2}\right)\log_e \frac{8a}{R} - \left(2 + \frac{R^2}{16a^2}\right)\right\} \quad . \quad (26)$$

where a is the radius of the circle to the centre of the wire, and R is the geometrical mean distance of the cross-section.

For wire of circular section, radius r

$$R = {\cdot}7788r$$

for a thin circular tube $\qquad R = r$

for a rectangular section $h \times k$

$$R = {\cdot}2235(h \times k)$$

for thin tape width $x \qquad R = {\cdot}2231x$

For circular wire and high frequencies use $R = r$.

For low frequencies or continuous current the most accurate formula for ordinary round wire is that given by Rayleigh and Niven :—

$$L = 4\pi a\left\{\left(1 + \frac{r^2}{8a^2}\right)\log_e \frac{8a}{r} + \frac{r^2}{24a^2} - 1{\cdot}75\right\} \quad . \quad . \quad (27)$$

The following are useful approximate formulas :—

For low frequencies

$$L = 4\pi a\left(\log_e \frac{8a}{r} - 1{\cdot}75\right) \quad . \quad . \quad . \quad . \quad (28)$$

(Kirchoff's formula.)

For high frequencies

$$L = 4\pi a\left(\log_e \frac{8a}{r} - 2\right) \quad . \quad . \quad . \quad . \quad (29)$$

EXAMPLES.—A. Inductance of a circle.

Diameter 1 foot. No. 6 S.W.G. wire.

Radius of circle $= 15{\cdot}24$ cms.

,, wire $= {\cdot}2438$,,

I. For continuous current formula (28)

$$L = 4\pi 15{\cdot}24\left(\log_e \frac{8 \times 15{\cdot}24}{{\cdot}2438} - 1{\cdot}75\right)$$

$$= 4\pi 15{\cdot}24(\log_e 500 - 1{\cdot}75)$$

$$= 191{\cdot}5(6{\cdot}2145 - 1{\cdot}75) = 854{\cdot}9 \text{ cms.}$$

II. For high-frequency currents, formula (29)

$$L = 191{\cdot}5(6{\cdot}2145 - 2) = 807{\cdot}1 \text{ cms.}$$

Inductance of a circle.

Diameter 1 foot = 30·48 cms. No. 18 S.W.G. wire, diameter ·12192 cm.

I. For continuous current

$$\begin{aligned} L &= 4\pi 15{\cdot}24(\log_e 2000 - 1{\cdot}75) \\ &= 191{\cdot}5(7{\cdot}6009 - 1{\cdot}75) \\ &= 191{\cdot}5(5{\cdot}8509) = 1120 \text{ cms.} \end{aligned}$$

II. For high-frequency currents

$$L = 191{\cdot}5(5{\cdot}6009) = 1073 \text{ cms.}$$

B. Inductance of a square of 1 foot side of (*a*) No. 6 S.W.G. wire, (*b*) No. 18 S.W.G. wire.

(*a*) I. For continuous current by formula (25)

$$\begin{aligned} L &= 8 \times 30{\cdot}48\left(\log_e \frac{30{\cdot}48}{{\cdot}2438} + \frac{{\cdot}2438}{30{\cdot}48} + 1{\cdot}226 - 1{\cdot}75\right) \\ &= 243{\cdot}84(\log_e 125 + {\cdot}0080 + 1{\cdot}226 - 1{\cdot}75) \\ &= 243{\cdot}8(4{\cdot}8283 + {\cdot}0080 + 1{\cdot}226 - 1{\cdot}75) \\ &= 243{\cdot}8(6{\cdot}0523 - 1{\cdot}75) = 1050 \text{ cms.} \end{aligned}$$

II. For high-frequency currents

$$L = 243{\cdot}8(6{\cdot}0523 - 2) = 988{\cdot}6 \text{ cms.}$$

(*b*) For the square of No. 18 S.W.G. wire.

For continuous current

$$\begin{aligned} L &= 243{\cdot}8(\log_e 500 + {\cdot}0020 + 1{\cdot}226 - 1{\cdot}75) \\ &= 243{\cdot}8(7{\cdot}4426 - 1{\cdot}75) = 1389 \text{ cms.} \end{aligned}$$

For high-frequency currents

$$L = 243{\cdot}8(7{\cdot}4426 - 2) = 1328 \text{ cms.}$$

The ratios of the inductance of a square to that of the inscribed circle as given by the four examples above is :

		Continuous Current.	High Frequency Current.
No. 6 S.W.G. wire	..	1·227	1·225
No. 18 S.W.G. wire	..	1·240	1·238

INDUCTANCE OF A COIL.

The inductance of a coil is the most important case in general work.

The most usual form is that of a coil wound on a cylindrical former, either in a single layer or in multiple layers.

The coil may have either an open core (air core) or an iron core, but in the latter case the calculation of the inductance from theoretical considerations alone, presents considerable difficulties, as explained on page 27.

The former, instead of being cylindrical, may be polygonal, square, or it may form part of the surface of a sphere.

Coils without Iron Cores.

Single-layer coils.

The case of a single-layer coil wound on a cylindrical former is one which occurs very frequently, and a large number of formulas have been given to suit the various ratios of length to diameter, etc.

Prof. Nagaoka has, however, given the following, which not only meets all possible ratios likely to be met with in practice, but gives the inductance to a high degree of accuracy with much less work than is required by several of the other formulas.

The formula is

$$L = \pi^2 d^2 n^2 l K \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where d = diameter of coil in centimetres,
l = length ,, ,,
n = number of turns per centimetre,

and K is a factor which depends on the ratio of the diameter to the length of the coil.

If the total number of turns of wire be used instead of the number of turns per centimetre, the formula may be written

$$L = \frac{\pi^2 d^2 N^2}{l} K \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where N is the total number of turns, and the other quantities are as above.

It will be seen that $\pi d N$ is the total length of wire on the coil, so that a third form is

$$L = \frac{x^2}{l} K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where $x = \pi d N$

The value of K for the ratios of length to diameter of the coil between ·01 and 10 have been calculated by Prof. Nagaoka to six

figures, and are given in the Bulletin of the Bureau of Standards, Vol. 8, No. 1.

The values of K to four figures will be found in Table 1, page 121.

For a coil of few turns Rayleigh and Niven's formula No. (27) may be used if the expression be multiplied by N^2, where N is the total number of turns on the coil.

Table for approximate values.

For making approximate calculations of the inductance of a coil, Table 2 has been worked out. This table gives the inductance of coils of diameters ranging from 4 to 18 cms. in diameter, and from 1 to 34 cms. long.

The inductance given is for a winding of 10 turns per centimetre. For other windings the values given in the table should be multiplied by $\frac{N^2}{100}$, where N is the number of turns per centimetre.

In Table 3 are given the number of turns per centimetre for coils wound with various covered wires. These figures have been worked out from the diameters given in the catalogue of a leading wire manufacturer, and in many cases checked on actual coils.

But they must be regarded as approximate only, since the number of turns per centimetre depends on the skill of the winder and the overall dimensions of the covered wire, which is not absolutely constant.

Subject to these limitations, the table will be found to give very good results for approximate work.

Correction for spacing between turns.

The inductance given by the above formulas is known as the "current sheet" value. That is, it is correct for a coil wound with thin flat strip, the turns of which touch without making contact, *i.e.* the insulation between them is infinitely thin.

For coils wound with ordinary round wires which have insulation of appreciable thickness, or which are spaced from one another, a correction must be made to obtain accurate results.

When the insulation is not thick compared with the conductor, and the coil has a large number of turns, as for the majority of wireless telegraphic receiving coils, this correction is small, but for other cases it is appreciable.

To make this correction, having calculated the inductance from the formula, subtract the quantity

$$\delta = 2\pi \mathrm{d n} l[\mathrm{A} + \mathrm{B}] \quad . \quad . \quad . \quad . \quad . \quad (33)$$

from the result, where n is the number of turns per centimetre.

The term A depends on the ratio $\frac{d_0}{D_1}$, where d_0 = diameter of the bare wire, and D_1 is the diameter over the insulation, or for coils wound with open turns, the distance between centres of the consecutive turns.

$\frac{d_0}{D_1}$.	A.	$\frac{d_0}{D_1}$.	A.
1·00	+·5568	·55	−·0410
·95	+·5055	·50	−·1363
·90	+·4515	·45	−·2416
·85	+·3943	·40	−·3594
·80	+·3337	·35	−·4928
·75	+·2691	·30	−·6471
·70	+·2001	·25	−·8294
·65	+·1261	·20	−1·0526
·60	+·0460	·15	−1·3404
		·10	−1·7457

The term B depends on the total number of turns on the coil.

No. of turns.	B.	No. of turns.	B.
1	+·0000	40	+·3148
2	+·1137	50	+·3186
3	+·1663	60	+·3216
4	+·1973	80	+·3257
6	+·2329	100	+·3280
8	+·2532	200	+·3328
10	+·2664	300	+·3343
15	+·2857	400	+·3351
20	+·2974	1000	+·3365
30	+·3083		

Multiple layer coil.

The correction to reduce the inductance to that for round insulated wire in the case of a coil which has a rectangular cross-section is as follows :—

$$L = L_u + \triangle L$$

where

$$\triangle L = 4\pi r n\left(\log_e \frac{D}{d} + \cdot 13806 + E\right) . \quad . \quad . \quad (34)$$

r being the mean radius of the coil, and d and D the diameters of the bare and covered wires respectively, and L_u the inductance given by the formula.

The term ·13806 reduces from a square to a circular cross-section for the conductor. The third term E is to correct for the difference between the mutual inductance for square wire with no space occupied by the insulation, to the actual case of insulated round wire.

Dr. E. B. Rosa has shown that E is not a constant, as first stated by Maxwell. Its value for certain cases has been given by him as follows, from which its value for any particular case can be obtained :—

Turns.	Layers.	E.
2	1	·00653
3	1	·00905
4	2	·01691
4	1	·01035
8	2	·01335
10	1	·01276
20	1	·01357
16	4	·01512
100	10	·01713
400	20	·01764
1000	20	·01778
infinite		·01806

Inductance of a Flat Spiral.

The inductance of a flat spiral of n turns may be calculated approximately as equal to that of a cylindrical coil of the same number of turns, with a diameter equal to the mean diameter of the spiral, and axial length equal to the width of the winding of the spiral, *i.e.* $l = R_1 - R_2$, where R_1 and R_2 are the radii of the outer and inner turns.*

Coil on Square Former.

A convenient form in which to wind a coil for many purposes is to make the former of square section.

There is no formula for this shape of coil, and moreover it is necessary to round the corners of the square to avoid sharp bends in the wire. The ratio of the radius at the bend to the side of the square may vary between wide limits, so that a large number of formulas would be required to meet every case.

* "The Principles of Electric Wave Telegraphy," J. A. Fleming, 2nd edition, p. 139.

An approximate value may be obtained by calculating the inductance of a circular coil of the same length and with a diameter equal to the length of the side of the square, and multiplying the result by the ratio of the inductance of a square to the circle of the given diameter.

This ratio is 1·23 . . . . . . (35)

as will be seen from the examples on page 17.

Inductance of Multiple Layer Coils.

This is an important case in practice.

Brooks and Turner's Formula.

This is an empirical formula, and gives results to a good approximation for any *closely-wound* coil, long or short, thick or thin, from a long solenoid to a single turn.

$$L = \frac{4\pi^2 a^2 N^2}{b+c+R} F_1 F_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

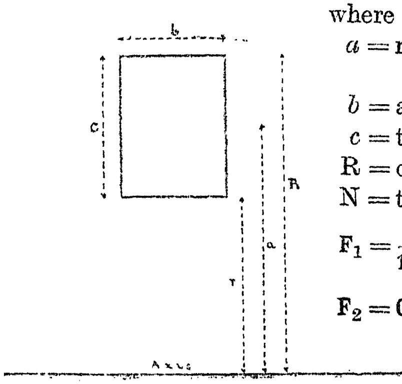


FIG. 1.

where

a = mean radius of the winding (see Fig. 1).

b = axial length of the coil,

c = thickness of the winding,

R = outer radius ,,

N = total number of turns.

$$F_1 = \frac{10b + 12c + 2R}{10b + 10c + 1{\cdot}4R}$$

$$F_2 = 0{\cdot}5 \log_{10}\left(100 + \frac{14R}{2b + 3c}\right) \quad (37)$$

For long coils F_1 and F_2 may for a first approximation be considered equal to unity, so that the formula reduces to its first term.

This formula is not suitable for open or spaced windings.

Butterworth's Formula.

The following formula has been given by Mr. Butterworth,* which he states will give an accuracy of four figures in the final result.

* Butterworth, "On the Self-Inductance of Solenoids of Appreciable Winding Depth," *Proc. Phys. Soc.*, xxvii. p. 381, June, 1915.

It is suitable for a coil with a length greater than twice the diameter, and a thickness of winding of less than one-tenth the diameter.

(1) Calculate the inductance of the coil from the following formula, which neglects the correction due to the ends :—

$$L_1 = 4\pi^2 R^2 N^2 l \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where N = number of turns *per unit length* including all layers.

(2) Apply the end correction to obtain the inductance L_2 for a current sheet of the mean radius of the coil from the formula

$$L_2 = L_1\left(1 - \frac{8}{3\pi c} + \frac{1}{2c^2} - \frac{1}{4c^4}\right)^* . \quad . \quad . \quad (39)$$

where $C = \dfrac{l}{R}$.

(3) Apply the thickness correction to obtain the inductance of a coil of winding depth $d = 2tR$

$$L_3 = L_2 + \triangle L_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where $$\triangle L_2 = -\tfrac{2}{3}L_1 t\left\{1 - \frac{t}{2} - \frac{t}{\pi c}\left(\log_e \frac{4}{t} - \frac{23}{12}\right)\right\} \quad . \quad . \quad (41)$$

(4) Apply the correction for the insulation space given by formula (34).

Perry's Formula.

Prof. Perry has given the following empirical formula, which is suitable for a short coil of which the length and width are small compared with the radius.

It is

$$L = \frac{4\pi n^2 a^2}{\cdot 2317a + \cdot 44b + \cdot 39c} \quad . \quad . \quad . \quad . \quad (42)$$

where n = whole number of turns on the coil,
a = the mean radius in cms.,
b = axial breadth,
c = radial depth.

EXAMPLE.—I. Calculate the inductance of a coil of 168 turns, outside diameter 27 cms., inside diameter 12·6 cms., axial length 2·0 cms.

$$R = \tfrac{27}{2} = 13{\cdot}5 \text{ cms.}$$
$$a = 9{\cdot}9 \;\;,,$$
$$b = 2{\cdot}0 \;\;,,$$
$$c = 7{\cdot}2 \;\;,,$$

* The value of this term is the same as the constant K in Nagaoka's formula, No. (30), which is given in Table 1.

By Brooks and Turner's formula

$$L = \frac{4\pi^2(9\cdot9)^2(168)^2}{2 + 7\cdot2 + 13\cdot5}F_1F_2 = 4810F_1F_2 \text{ microhenrys}$$

$$F_1 = \frac{2 \times 10 + 12 \times 7\cdot2 + 27\cdot0}{2 \times 10 + 10 \times 7\cdot2 + 1\cdot4 \times 13\cdot5} = \frac{133\cdot4}{110\cdot9} = 1\cdot203$$

$$F_2 = \cdot5 \log_{10}\left(100 + \frac{14 \times 13\cdot5}{4 + 21\cdot6}\right)$$

$$= \cdot5 \log_{10}\left(100 + \frac{189}{25\cdot6}\right)$$

$$= \cdot5 \log_{10} 107\cdot38 = 1\cdot015$$

Hence $L = 4810 \times 1\cdot203 \times 1\cdot015 = 5873$ microhenrys

By Perry's formula

$$L = \frac{4\pi(168)^2(9\cdot9)^2}{\cdot2317 \times 9\cdot9 + \cdot44 \times 2 + \cdot39 \times 7\cdot2}$$

$$\frac{34760 \times 10^3}{2\cdot294 + \cdot88 + 2\cdot808} = \frac{34760 \times 10^3 \text{ cms.}}{5\cdot982}$$

$$= 5817 \text{ microhenrys}$$

EXAMPLE.—II. Calculate the inductance of a coil 12 cms. long, mean diameter 10 cms., depth of winding ·8 cm., wound with 20 turns per cm. in four layers No. 15 S.W.G. wire.

Total number of turns $= 240 = N$

mean radius $= 5$ cms. $= r$

diameter of wire $= \cdot183$ cm.

By Butterworth's formula No. (39)

$$L_2 = \frac{4\pi^2N^2r^2}{l}\left(1 - \frac{8}{3\pi c} + \frac{1}{2c^2} - \frac{1}{4c^4}\right)$$

where $c = \dfrac{\text{length}}{\text{radius}} = 2\cdot4$.

$$L_2 = \frac{4\pi^2(240)^2(5)^2}{12}\left(1 - \frac{8}{3 \times 2\cdot4 \times \pi} + \frac{1}{2(2\cdot4)^2} - \frac{1}{4(2\cdot4)^4}\right)$$

$$= 4737 \times 10^3(1 - \cdot3537 + \cdot0888 - \cdot0075)$$

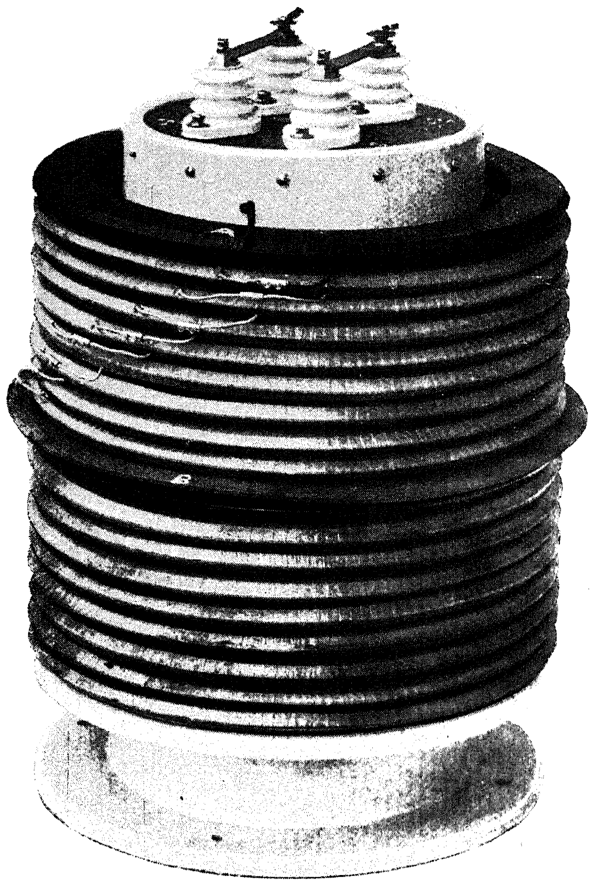
$$= 4737 \times 10^3 \times \cdot7276 \text{ cms.} = 3446 \text{ microhenrys.}$$

Correction $\triangle L_2$ for depth of winding

$$-\frac{2}{3}L_1T\left\{1 - \frac{T}{2} - \frac{T}{\pi c}\left(\log_e\frac{4}{T} - \frac{23}{12}\right)\right\}$$

$$T = \frac{\text{depth}}{\text{diameter}} = \cdot08$$

$$-\frac{2}{3} \times 4737 \times 10^3 \times \cdot08\left\{1 - \cdot04 - \frac{\cdot08}{\pi 2\cdot4}\left(\log_e 50 - 1\cdot9166\right)\right\}$$



L.F. Secondary Tuning Inductance, 5 K.W.

(Page 25.

$$-\frac{\cdot16\times4737}{3}\left\{\cdot96-\frac{1}{30\pi}\left(3\cdot9120-1\cdot9166\right)\right\}$$

$$=4737\times\frac{\cdot16}{3}(\cdot9388)\times10^3 \text{ cms.}$$

$= 237\cdot2$ microhenrys

$= 3446 - 237 = 3209$ microhenrys

Correction for insulation of wire, etc.

$$=4\pi a\mathrm{N}\left(\log_e\frac{\mathrm{D}}{d}+\cdot1381+\mathrm{E}\right)$$

$$=4\pi\times5\times240\,(\log_e 1\cdot09+\cdot1381+\cdot0173)$$

$= 3\cdot6$ microhenrys

Inductance $= 3212$ microhenrys

Inductance by Brooks and Turner's formula

$$\mathrm{L}=\frac{4\pi^2(5)^2240^2}{12+\cdot8+5\cdot4}\left\{\frac{120+9\cdot6+10\cdot8}{120+8\cdot0+7\cdot56}\right\}\left\{\frac{1}{2}\log_{10}\left(100+\frac{75\cdot6}{24+2\cdot4}\right)\right\}$$

$$\frac{100\pi^2\times240^2}{18\cdot2}\left\{\frac{140\cdot4}{135\cdot6}\right\}\left\{\frac{1}{2}\log_{10}\left(102\cdot8\right)\right\}$$

$= 3123(1\cdot038)(1\cdot006)$ mhy.

$= 3258$ microhenrys

Inductance of Built-up Coils.

The inductance is often required of a coil formed by joining a number of coils in series, their axes being in the same line. If the coils are packed tight together, and the outer insulation over each coil be thin, then the inductance might be calculated from the formulas given above.

In many cases the coils are separated by spacing pieces from one another and, moreover, the coils need not be identical as regards number of turns. Thus in the coil shown on page facing, the complete unit comprises two half-sections, and each half-section has eight coils, with identical numbers of turns and an upper coil with considerably fewer turns.

We will consider the simplest case first—that of a built-up inductance of a number of identical coils equally spaced. The inductance of this can be found approximately by calculating the inductance of a coil of equal axial length and with the mean diameter of the actual coils, assuming each unit coil to be replaced by a single turn.

This may be obtained by Nagaoka's formula, with the proper corrections for spacing. The section of the single turn may be taken to be that of the cross-section of the unit.

The resulting inductance is multiplied by n^2, where n is the number of turns in each unit.

The formula is therefore

$$L = 4\pi^2 m^2 n^2 R^2 l + 4\pi r m[A + B] \quad . \quad . \quad . \quad (43)$$

where m = no. of units,
n = number of turns per unit,

and the correction term is for the equivalent single layer coil.

A more accurate method, and one which is applicable to coils with unequal numbers of turns, is to calculate the self-inductances of the separate coils and add twice the sums of the mutual inductances between each coil and all the others.

$$L = L_1 + L_2 \ . \ . \ . + Ln + 2(\Sigma M_1 \ . \ . \ . + \Sigma Mn) \quad (43a)$$

where L_1 = self-inductance of coil 1,
ΣM_1 = sum of the mutual inductances between coil 1 and all the others.

If the coils are all equal

$$L = nL_1 + 2(n-1)M_{12} + 2(n-2)M_{13} + \text{etc.} \quad (43b)$$

M_{12} = mutual inductance between coils 1 and 2
M_{13} = ,, ,, ,, coils 1 and 3, etc.

The mutual inductances can be obtained by the method of formula (57).

Coil of Maximum Inductance.

In winding an inductance it is desirable in most cases to make its resistance as small as possible, since in this case the length of wire will also be a minimum. It is not possible to give a single formula for finding the dimensions of a coil with maximum inductance in every case, since the value of the inductance depends on the correction terms for thickness of insulation, etc., given in formulas (33) and (34). The original formula given by Maxwell is that if the section of the coil is square the maximum inductance is obtained when the mean diameter of the coil is 3·7 times the side of the square.

In a recent article Messrs. Shawcross and Wells * have given curves of the inductance obtained for a wire 1 mm. in diameter and 1570·8 (500π) metres long coiled up in different forms. The coil of maximum inductance for a square section coil is that from the curves with diameter about three times the side of the square.

* *Electrician*, April 16, 1915.

The square section coil has a greater inductance than coils of other ratios of width b to depth c.

Thus the inductance for the square section coil is 1·29 henrys (approx.).

For a ratio $\frac{b}{c} = 2$ the maximum inductance is 1·24 henrys, and is for a ratio $\frac{d}{b}$ about 2·3.

For $\frac{b}{c} = 5$ max. inductance $= 1{\cdot}07$ henrys ; for $\frac{d}{b} = 1{\cdot}5$.

Dr. A. Russell * has pointed out that for the maximum possible inductance from a given length of wire the section of the coil should be circular, and with a ratio $a = 2{\cdot}575r$, where a is the radius of the circular axis of the coil, and r is the radius of the cross-section.

The inductance is

$$L = 5{\cdot}35\pi N^2 a \times 10^{-9} \text{ henrys}$$

where N = total number of turns.

For a single-layer coil for the case where the turns are close-wound, the coil of maximum inductance is given by $\frac{\text{diameter}}{\text{breadth}} = 2{\cdot}415$.

This can be seen by plotting a curve of the inductance as calculated by Nagaoka's formula No. (30).

When the coil is wound with spaced windings the inductance will depend on the spacing, and therefore it must be worked out for any particular case from the formulas given.

INDUCTANCE OF COILS WITH IRON CORES.

The formulas for inductance given in the previous formulas have been for coils without magnetic cores.

The inductance of these coils depends principally on the geometrical dimensions, with a small correction factor depending on the frequency of the current, so that there are formulas available for every possible case, many being of the highest accuracy.

For coils with iron cores, however, the inductance depends on the physical properties of the core as well as the dimensions of the coil, and these properties are variable. The flux density B, which is induced in a sample of iron by a magnetising force H, is given by the relationship

$$B = \mu H$$

where μ is the permeability.

* *Electrician*, April 23, 1915.

Now, μ is not a constant for any one sample, but varies in a complex manner with H, and also varies within wide limits for different samples of iron.

The inductance of an iron cored coil, therefore, depends on the permeability of the iron, and this varies over the cross-section of the core, since H is not constant over that area. Moreover, if the current in the coil (by which H is produced) varies, the value of μ will change.

It is, therefore, only possible to select some average value for μ which can be taken by experience to give a representative value for the particular coil in question.

Where the coil is in such a form that the iron forms a nearly closed circuit with a relatively small air gap, a convenient formula is

$$L \text{ (henrys)} = \frac{\cdot 4 N^2 \mu A}{l} \quad . \quad . \quad . \quad . \quad (44)$$

in which μ is the permeability of the air gap ($\mu = 1$),

l is the length of the air gap in centimetres,

A is the cross-section normal to the flux at the gap (the area of the surface of the iron core at the air gap),

N is the total number of turns on the coil.

For an open-core coil a similar formula which may be used is

$$L = \frac{N^2 A \mu}{l} k \quad . \quad . \quad . \quad . \quad . \quad . \quad (44a)$$

where N = total number of turns,

l = length of the iron core,

A = cross-section of the core,

μ = the permeability for the value of H at the centre of the core,

and k is a factor to be determined by experiment or from previous data to allow for the effect of the ends.

The area A is strictly that of the iron in the core after allowing for air spaces or insulation between the strips or wires of which the iron is composed, but if the same iron be used for various coils the outside area of the core is taken and the difference allowed for in k.

Mutual Inductance of Two Circles.

The mutual inductance between two coaxial circles is important, as it affords a simple method for obtaining the mutual inductance between two coils.

There are several formulas for obtaining the mutual inductance between two circles, some suitable for circles near together and some for circles at some distance apart.

The most convenient, however, are two due to Prof. Nagaoka.

The first is

$$M = 16\pi^2\sqrt{Aa}\, q^{\frac{3}{2}}(1 + e) \quad . \quad . \quad . \quad . \quad (45)$$

where
$$q = \frac{l}{2} + 2\left(\frac{l}{2}\right)^5 + 15\left(\frac{l}{2}\right)^9 + \ldots$$

$$l = \frac{1 - \sqrt{k'}}{1 + \sqrt{k'}}$$

$$k' = \frac{R_2}{R_1} = \frac{\sqrt{(A - a)^2 + d^2}}{\sqrt{(A + a)^2 + d^2}}$$

$$e = 3q^4 - 4q^6 + 9q^8 - 12q^{10} \ldots$$

A and a are the radii of the two circles respectively.

d is the distance between their centres.

This formula will be found most suitable when the ratio $\frac{R_2}{R_1}$ is greater than 0·3 (see Fig. 2).

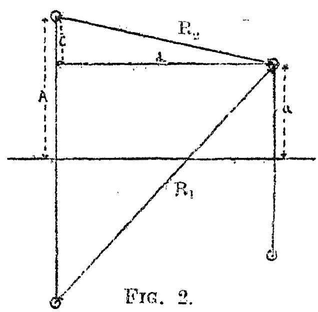


FIG. 2.

To facilitate calculations with this formula, tables have been worked out giving the values of the difference between q and $\frac{l}{2}$, and also of e and $\log_{10}(1 + e)$, for values of q between ·02 and ·15.

These tables appear in the Bulletin of the Bureau of Standards, Vol. 8, No. 1.

It is to be noted that the difference between q and $\frac{l}{2}$ for $q = {}$·15 is only ·000152 and of $\log_{10}(1 + e)$ is only ·00064, so that this term may be omitted and q taken equal to $\frac{l}{2}$, where an accuracy of about 1 in 500 is alone required.

Nagaoka's second formula is of a similar form, but some of the terms have different meanings. It is

$$M = 16\pi^2 q'^{\frac{3}{2}}(1 + e') \quad . \quad . \quad . \quad . \quad . \quad (46)$$

where
$$q' = \frac{l'}{2} + 2\left(\frac{l'}{2}\right)^5 + 15\left(\frac{l'}{2}\right)^9 + \dots$$
$$l' = \frac{1-\sqrt{k_2}}{1+\sqrt{k_2}} = \frac{k_1^2}{(1+k_2)(1+\sqrt{k_2})^2}$$
$$k_1 = \frac{R_1 - R_2}{R_1 + R_2} = \frac{4Aa}{(R_1+R_2)^2}$$
$$k_2 = \frac{2\sqrt{R_1R_2}}{R_1+R_2}$$
$$R_1 = \sqrt{(A+a)^2 + d^2}$$
$$R_2 = \sqrt{(A-a)^2 + d^2}$$
$$e' = 3q^2 - 4q^3 + 9q^4 - 12q^5$$

In this case the term $(1+e)$ is of more importance, being 1·0572 for $q' = \cdot 15$.

But the difference between q' and $\frac{l'}{2}$ is the same as for the previous formula, so that the terms $\left(\frac{l}{2}\right)^5$, etc., may be omitted as before.

The values of $\log_{10}(1+e')$ are not given in the Bureau of Standards Bulletin, quoted above, but will be found in the Proceedings of the Physical Society of London, vol. xxv. p. 31.

It is not necessary to quote any of the other formulas for the mutual inductance of circles beyond the following :—

$$M = 4\pi a\left(\log\frac{8a}{r} - 2\right) \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where a = radius of the smaller circle,
r = the wire distance $\sqrt{c^2+d^2}$, where
d = axial distance between the circles,
$c = A - a$.

This formula is suitable for circles which are near together and of nearly the same diameter.

Tables for the Mutual Inductance of Circles.

As the mutual inductance of circles is useful in working out the mutual inductance of coils, the author has worked out the value of M_0 for circles of unit radius for various values of $\frac{R_2}{R_1}$, where R_2 and R_1 are the least and greatest distance between the circles (see Fig. 2). The values are given in Table 4.

For circles of radii A and a respectively the mutual inductance is

$$\mathbf{M} = \mathbf{M}_0 \sqrt{\mathbf{Aa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Tables giving M_0 in terms of an angle γ where $\sin \gamma = \sqrt{\frac{R_1{}^2 - R_2{}^2}{R_1}}$ will be found in the Bulletin of the Bureau of Standards, vol. 8, No. 1.

See also "A Treatise on Electricity" by Pidduck, page 197.

Mutual Inductance of Linear Conductors.

The mutual inductance of linear conductors of various shapes is required in calculating the total inductance of a pair of leads, such as the connections to a piece of apparatus.

For a pair of round parallel wires

$$M = 2\left(l \log_e \frac{l + \sqrt{l^2 + d^2}}{d} - \sqrt{l^2 + d^2} + d\right) . \quad (49)$$

where $\quad l =$ length of either wire,

$\quad d =$ distance between them,

where d can be neglected in comparison with l.

$$M = 2l\left(\log_e \frac{2l}{d} - 1 + \frac{d}{l}\right) . \quad . \quad . \quad . \quad . \quad (50)$$

For two parallel tapes or rectangular bars

$$M = 2l\left(\log_e \frac{2l}{R} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (51)$$

where R = G.M.D. of one tape from the other (see page 14), formula (21).

Mutual Inductance of Rectangles.

The only useful formula is that for two equal rectangles parallel to one another, the equal sides being parallel.

The formula, due to Neumann, is

$$M = 4\left(a \log_e \frac{(a + x)y}{(a + z)d} + b \log_e \frac{(b + y)x}{(b + z)d}\right)$$
$$+ 8(z + d - x - y) . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

where

$$x = \sqrt{a^2 + d^2}$$
$$y = \sqrt{b^2 + d^2}$$
$$z = \sqrt{a^2 + b^2 + d^2}$$

a and b being the sides of the rectangles, and d their distance apart.

Mutual Inductance of Two Coaxial Coils.

The mutual inductance between two coaxial coils which have a rectangular cross-section is usually obtained by first calculating the mutual inductance between certain equivalent circles.

An approximate formula for the case where the distance between the coils is large compared with the diameter is

$$M = n_1 n_2 M_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

where M_x is the mutual inductance between the two central turns of the coils, and n_1, n_2 are the number of turns on the coil. M_0 is given by formulas (45) and (46).

A more exact formula has been given by Rayleigh.

$$M = \tfrac{1}{6}(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 - 2M_0) \qquad (54)$$

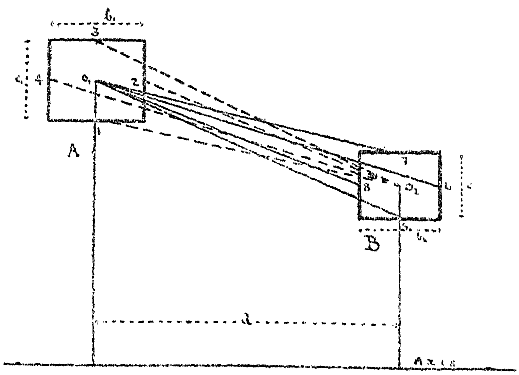


FIG. 3.

where M_1 is the mutual inductance of the circle through O_2 and a circle through the point 1, and similarly for M_2, etc. (see Fig. 3).

For two coils of equal radii and equal section this becomes

$$M = \tfrac{1}{3}(M_1 + M_2 + M_3 + M_4 - M_0) \quad . \quad . \qquad (55)$$

The error in these formulas is greater as the ratio of length to axial depth increases, and thus they are not suitable for single-layer coils unless they are very short and at a great distance apart.

Lyle's Formula.

Prof. Lyle has given a very convenient method for the mutual inductance between two coaxial coils which is very accurate when the coils are at some distance apart.

Each coil is replaced by two equivalent circles (see Fig. 4).

The mutual inductances for the four pairs of circles AC, AD, BC, BD, are calculated, and the arithmetic mean taken. This gives the mutual inductance of the coils.

Hence

$$M = \tfrac{1}{4}\{M(AC) + M(AD) + M(BC) + M(BD)\} \quad (56)$$

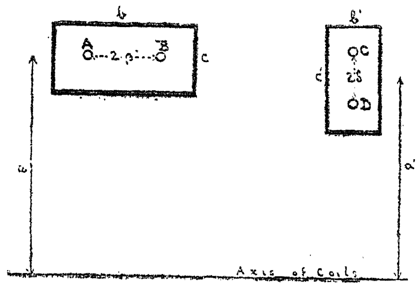


FIG. 4.

The equivalent radius of the circles A and B (for a coil with the breadth greater than the depth) is given by

$$r_1 = a\left(1 + \frac{c^2}{24a^2}\right)$$

a being the mean radius.

The radius of circles C and D (for a coil with the depth greater than the breadth) is given by

$$r_C = r_3 + d$$

$$r_D = r_3 - d$$

where

$$r_3 = a'\left(1 + \frac{b'^2}{24a'^2}\right)$$

and

$$\delta^2 = \frac{c'^2 - b'^2}{12}$$

The distance between A and B is given by 2β, where

$$\beta^2 = \frac{b^2 - c^2}{12}$$

(see Fig. 4).

Mutual Inductance between Single-Layer Coils.

There are a number of formulas for the mutual inductance of single-layer coils, the particular one to use depending on whether the coils are concentric or at a distance apart, or whether the inner is shorter than the outer, etc.

These formulas are mostly very complicated, and if, for example, a curve of the mutual inductance between two coils for various relative positions were required, the work involved would be very great.

Author's Method.

The author has therefore devised the following graphic method, by which results of fair accuracy can be obtained with a small amount of work, and which is very suitable for plotting a curve of the nature referred to.

The method consists in dividing each coil into a number of equal sections, so that the axial length of each is short compared with its radius. The larger the number of sections taken the more work will be involved, with a corresponding increase in accuracy (Fig. 5).

Each of these sections is replaced by a circle at its centre, and

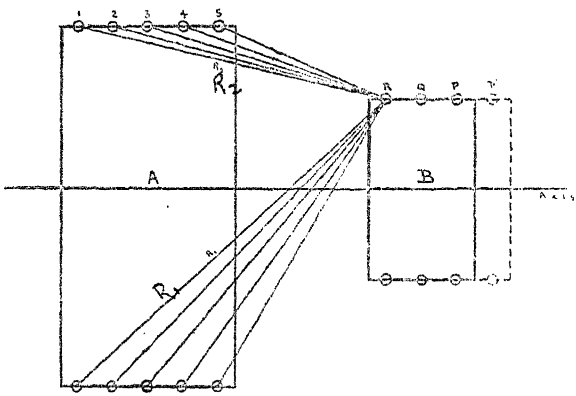


FIG. 5.

the mutual inductance between each of the equivalent circles of one coil and each of the circles for the other coil is obtained.

This value is obtained by measuring the ratio $\frac{R_2}{R_1}$ of the least and greatest distances between each pair of circles.

This can easily be obtained by making a scale drawing, and measuring with a millimetre scale, the ratio being obtained on a slide rule.

The mutual inductance is obtained for these values of $\frac{R_2}{R_1}$ from Table 4, page 125, and the mutual inductance between the coils is given by

$$M_{AB} = n_1 n_2 \sqrt{Aa}(M_x) \quad . \quad . \quad . \quad . \quad (57)$$

where M_x is the arithmetic mean of all the mutual inductances between pairs of coils and A, a the respective radii.

If a curve be required of the variation in mutual inductance between the coils for various relative distances, having obtained the value at one position as above, one coil is moved through a distance equal to the length of one section.

Thus in Fig. 5 the coil B is moved so that the circle P moves to P′, Q moves to P, and R to Q.

It is therefore only necessary to measure the ratio $\frac{R_2}{R_1}$ for the new position P′, those for the new positions of R and Q being already obtained for the first result.

When the coils are at a considerable distance apart, the work may be shortened by replacing each by two equivalent circles given by Prof. Lyle's formula, No. (56).

The method can, of course, be used for multiple-layer coils of which the depth of winding is not too great compared with the radius.

It may be used for obtaining the self-inductance of a built-up coil, as mentioned on page 25.

When the lengths of the two coils are in a simple ratio, the determination of the mutual inductance by this method is greatly simplified. The coils should be divided into a convenient number of sections which are equal in length for both coils.

The various equivalent circles will then be spaced at equal distances apart, so that a determination of $\frac{R_2}{R_1}$ for one set of coils will give the values for all sets.

EXAMPLE.—The mutual inductance between the following coils, when the coils are placed with their nearer faces in one plane, was calculated by formula (57).

Coil No. 1, diameter 16·8 cms., length 17·1 cms., wound with 212 turns.

Coil No. 2, diameter 12·3 cms., length 6·6 cms., wound with 80 turns.

A diagram (full size) was drawn, and the first coil was divided into eight equal sections, and the second coil into five sections. Each section was replaced by the circle through its centre, and the $8 \times 5 = 40$ sets of ratios of $\frac{R_2}{R_1}$ measured.

Thus, for the first circles of each coil the distances R_2 and R_1 are 16·8 and 22·15 cms., the ratio being $\frac{16\cdot8}{22\cdot15} = \cdot759$, for which M_0 from the table $= 1\cdot009$.

For the last section of coil 1 and the first section of coil 2 the ratio was $\frac{2\cdot80}{14\cdot62} = \cdot1912$, for which $M_0 = 13\cdot81$.

The sum of the 40 values for M_0 was 128·304.

Hence mutual inductance

$$= \frac{128\cdot30}{40} \times \sqrt{\frac{17\cdot1}{2}\,\frac{12\cdot3}{2}} \times 212 \times 80$$

$$= 391 \text{ microhenrys}$$

When measured by the Campbell Mutual Inductometer the mutual inductance was 382 microhenrys.

To obtain a curve of the mutual inductance for various positions, the smaller coil is moved so that its second circle coincides with the original position of the first circle, when only five new measurements are required, the first 35 of the previous set coinciding with the last of the new position, and so on.

Mutual Inductance between Two Coaxial Coils of the same Radius and Winding.

The mutual inductance between two coils of the same diameter, and having the same number of turns per centimetre, is of great importance, since by its means the total inductance of a series of coils wound on one former with spaces between them can be determined.

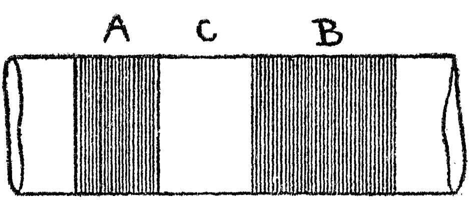


Fig. 6.

Let the mutual inductance of coils A and B (Fig. 6) be required.

Imagine the space between A and B to be filled by a coil C of the same number of turns per centimetre. Then the mutual inductance between A and B is given by

$$2M_{AB} = L_{ABC} + L_C - L_{AC} - L_{BC} \quad . \quad . \quad . \quad (58)$$

where L_{ABC} is the self-inductance of coils A, C, B in series, and so on.

These self-inductances can be worked out by Nagaoka's formula (30) or Tables 1 and 2.

REFERENCES.

A complete collection of formulas is given in the Bulletin of the Bureau of Standards (U.S.A.), vol. 8, No. 1. Reprint No. 169 by Rosa and Grover. The most suitable formulas for any particular case are indicated.

Dr. Eccles' "Handbook of Wireless Telegraphy and Telephony" has a large selection of formulas. Formulas Nos. (3), (11), and (24) are taken from this work.

Dr. Fleming's "Wireless Telegraphist's Pocket Book" gives a large number of formulas and worked examples.

CHAPTER II

THE CALCULATION OF CAPACITY

The definition of capacity is usually given as: "The capacity of a conductor is defined and measured by the quantity of electricity required to raise its potential from zero to unity." The capacity given by this expression is the capacity with reference to the body or bodies at the zero of potential from which the potential of the given conductor is raised by the charge.

If the conductor be at a great distance from all other bodies, then the capacity is the capacity in free space, but if it be near some other body, such as the earth or another conductor, the capacity will be the capacity to earth or between the two conductors.

A condenser consists of two sets of conductors placed so near each other that the capacity between them is large compared with the capacity of either (or both) to earth or surrounding objects.

The absolute unit on the electromagnetic system is the farad, and the practical unit the microfarad.

In calculating capacity, if all dimensions be in centimetres the capacity is given in electrostatic units or centimetres.*

Parallel Plate Condenser.

This is one of the most usual forms of condenser. It consists of two sets of metal armatures, consisting of one or a number of plates, placed close together, with or without a sheet of solid dielectric material between. In place of solid dielectrics between the plates, they may have either air or be immersed in an oil dielectric.

The capacity of a parallel plate condenser consisting of two metal armatures with a single sheet of dielectric between, is approximately given by the formula

$$C = \frac{KA}{4\pi d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where A = area of one metal plate in square centimetres,

d = distance between the plates (or thickness of the dielectric) in centimetres,

K = dielectric constant (specific inductive capacity) of the dielectric. For an air condenser $K = 1$.

* 1 microfarad = 9×10^5 centimetres.

The capacity given by this and the following formulas (except where otherwise stated) is in *electrostatic units* or centimetres.

The capacity of a multiple plate condenser is best obtained by considering the number of sheets of dielectric (air or otherwise) enclosed between the metal plates.

If this be n,

$$C = \frac{nKA}{4\pi d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Those formulas are approximate only as they neglect the "edge effect"; that is, it is assumed that the lines of electric stress between the met . plates are perpendicular to the surfaces with no curving at the ·dges, such as actually is the case.

If ··e condenser be provided with a guard ring, the formulas are correct. A guard ring consists of a metal ring or rim round the outside edge of each metal plate of the condenser, and separated therefrom by a narrow gap.

When both the guard ring and adjacent metal plate are charged to the same potential, the lines of stress for the condenser will be perpendicular to the metal plates even at the edges, the curved lines being at the outer edge of the guard ring. The capacity of the condenser is then given by the above formula exactly, provided that the guard ring remains always at the same potential as the metal condenser plate, *i.e.* is discharged, etc., at the same time as the plate through a separate circuit.

In using the above formulas it is to be noted that for most condensers with solid dielectric, the capacity will vary considerably with the pressure applied during process of manufacture. The pressure in some cases squeezes out any binding agent used to fix the metal plates on the dielectric sheets, or it may alter the thickness of the sheets themselves. The capacity of these condensers must therefore be measured after completion, and may be very different from the calculated value.

Similarly for condensers built of glass plates, the thickness of the glass varies to some extent between individual sheets and also between different parts of the surface of each sheet, so that it is not possible to measure the distance between the metal plates exactly.

The only formula which takes account of the "edge effect" is that due to Kirchoff for the capacity between two parallel circular plates, a form which is not often used in practice.

The formula is

$$C = \frac{\pi r^2}{4\pi d} + \frac{r}{4\pi d}\left[d\left\{\log_e \frac{16\pi r(d+t)}{d^2} - 1\right\} + t\log_e\left(1+\frac{d}{t}\right)\right] + C_0 \quad (3)$$

where r is the radius, t the thickness of the circular plates (in centimetres), and d is the distance between them. C_0 is the part of the capacity which does not depend on d (*i.e.* the capacity of connecting wires, etc.).

Sir J. J. Thomson has shown that in the case of a condenser with plates having straight edges, if the linear dimensions of the plates of a condenser are large compared with the distance between them, the "edge effect" may be allowed for by considering it as due to an additional strip of width w where

$$w = \frac{d}{2\pi}(1 + \log_e 2) = \cdot 110d \quad . \quad . \quad . \quad . \quad (4)$$

A similar formula for circular plate condensers has been given by Maxwell:

$$w = \frac{2d}{\pi}\log_e 2 = \cdot 4413d \quad . \quad . \quad . \quad . \quad (5)$$

Example.—Calculate the capacity between a pair of circular plates 20 cms. diameter, 2 mm. spacing, 1 mm. thick.

By formula (3)

$$C = \frac{\pi(10)^2}{4\pi \cdot 2} + \frac{10}{4\pi \cdot 2}\left[\cdot 2\left\{\log_e \frac{16\pi 10(\cdot 3)}{\cdot 04} - 1\right\} + \cdot 1 \log_e\left(1 + \frac{\cdot 2}{\cdot 1}\right)\right]$$
$$= \frac{100}{\cdot 8} + \frac{10}{\cdot 8\pi}[\cdot 2(\log_e 3770 - 1) + \cdot 1 \log_e 3]$$
$$= 125 + 3\cdot 979(\tfrac{1}{5} \times 7\cdot 2348 + \cdot 1099)$$
$$= 125 + 3\cdot 979(1\cdot 557) = 131\cdot 2 \text{ cms.}$$

If the thickness of the plates be negligible

$$C = 125 + 3\cdot 979[\tfrac{1}{5}(\log_e 2514 - 1)]$$
$$= 125 + 3\cdot 979[\tfrac{1}{5}(7\cdot 8294 - 1)]$$
$$= 130\cdot 46 \text{ cms.}$$

The capacity by formula (5) $= \frac{\pi(10\cdot 88)^2}{4\pi \cdot 2} = 148$ cms., which is 13 per cent. too high.

The capacity of a *sphere* at a very great distance from other conductors is given by

$$C = r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and of a *thin circular disc*

$$C = \frac{2r}{\pi} = \cdot 6364r \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Prof. Fleming, in "The Principles of Electric Wave Telegraphy," 2nd edition, page 179, quotes a measurement of the capacity of a disc 60 inches in diameter suspended in a room.

The calculated capacity was 53·44 micromicrofarads, and the measured capacity 59·95, which is about 10 per cent. greater.

Capacity between Spheres.

For two spheres radii r_1 and r_2 at a great distance a

$$C = \frac{r_1 r_2 a}{(r_1 + r_2)a - 2r_1 r_2} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The joint capacity of the two spheres is

$$C = a\left\{\frac{(r_1 + r_2)a - 2r_1 r_2}{a^2 - r_1 r_2}\right\} \quad . \quad . \quad . \quad . \quad (9)$$

For two equal spheres close together

$$C = \frac{1}{2} r\left(1 + \frac{x}{6r}\right)\left(1{\cdot}2704 + \frac{1}{2}\log_e \frac{r}{x} + \frac{x}{18r}\right) \quad . \quad (10)$$

and the joint capacity

$$C = r\left(1 + \frac{x}{6r}\right)\left(\cdot 3863 - \frac{x}{12r}\right) \quad . \quad . \quad . \quad (11)$$

where x is the distance between the nearest points. For an accuracy of ·1 per cent. x should be less than $\frac{1}{10}$ the radius.

(Russell.)

Variable Condensers.

The principal types of variable condensers are:

(1) A set of metal vanes are fixed to a spindle, by the rotation of which they are interleaved with a fixed set of vanes.

(2) A metal tube sliding over a cylindrical metal plug.

(3) A set of flat plates sliding rectilinearly over a similar set of plates.

Of the first type the commonest form is that in which both sets of plates are of semicircular shape with the central part of the fixed plates removed to make place for the spindle of the moving plates.

The capacity is given by the formula $C = \frac{KA}{4\pi d}$, which reduces to

$$C = Kn \frac{\pi(r_1^2 - r_0^2)}{4\pi d} \frac{\theta}{360} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where r_1 is the radius of the outside edge of the smaller of the sets of plates, r_0 the radius of the inside edge of the fixed plates, n

the number of air spaces or sheets of dielectric between the plates, and θ is the angle in degrees through which the plates have been turned from the position of minimum capacity. This formula neglects the edge effect.

The capacity of an actual condenser of this type is not strictly proportional to the angle through which the vanes have turned. The capacity at minimum is larger than that given by the formula. A calibration curve of such a condenser is shown in Fig. 7.

From this it will be seen that the difference in capacity for a

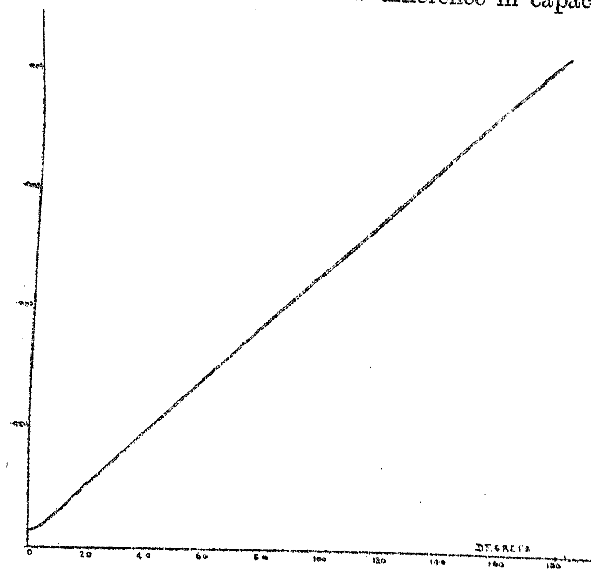


FIG. 7.

given change in angular displacement is proportional to the angular displacement except at two ends of the scale.

Square Law Condenser.

For many purposes condensers for which the variation in capacity for a given angular displacement follows some other law than the straight line above are required.

Mr. Duddell, in a recent paper,* gives a formula for a condenser in which the variation in capacity is proportional to the square of the angle through which the spindle is turned.

* *J.I.E.E.*, vol. 52, p. 275; *Electrician*, Feb. 6, 1914; *Wireless World*, April, 1914.

In the paper Mr. Duddell shows that there are a number of possible shapes for the plates, but that some are not suitable from a mechanical point of view.

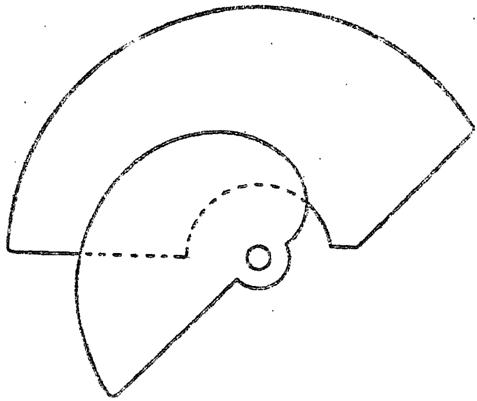


FIG. 8.

The type adopted by him is shown in Fig. 8, in which the inside edge of the fixed set of plates is a semicircle concentric with the

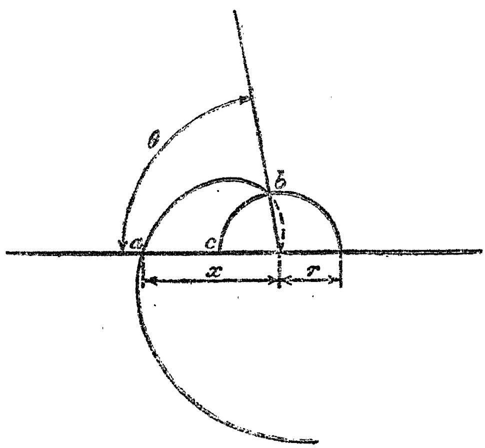


FIG. 9.

spindle of the moving plates, and the outside edge is so large that the moving plate never projects beyond it.

The curve of the outside edge of the moving plate is obtained as follows :—

Let r (Fig. 9) be the radius of the inside edge of the fixed plates,

and x be one of the radii of the curve forming the outside edge of the moving plate.

The formula for making the area pqr bounded between the curve and the circle of radius r proportional to the square of the angle θ is

$$x^2 = 4k\theta + r^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where k is a constant such that

$$k\theta^2 = \text{A}$$

where A is the area of the moving plates between the fixed plates.

To construct such a condenser a value for r is selected, and then k taken, so that the area A is such as to give the required capacity. From these values x is obtained from equation (13).

The edge effect has been neglected in the formula. In order to minimize it the radius r must not be too small.

The following details of an actual condenser given in Mr. Duddell's paper may be of use in designing a condenser of this type, the number of moving plates being 13, and of fixed plates 14. Clearance 1 mm. approx.

The radius r was first taken at 2 cms., and $4k$ at $\frac{36}{\pi}$, the angle being measured in radians, so that area $= \frac{9}{\pi}\theta^2$.

The equation of the curve is therefore

$$x^2 = \frac{36}{\pi}\theta + 4 \quad \text{for } \theta \text{ in radians, or}$$

$$x^2 = \cdot 2\theta + 4 \quad \text{for } \theta \text{ in degrees.}$$

When tested, the capacity of the condenser was found to be not quite proportional to θ^2. By increasing r to 2·3 cms. without altering the shape of the curve, the capacity of the condenser being represented by

$$\text{C} = a + b\theta + c\theta^2$$

where θ is in radians and c in millimicrofarads, the data for the two condensers above were

$r = 2$ cms.	$r = 2{\cdot}3$ cms.
$a = \cdot 022$	$a = \cdot 025$
$b = \cdot 017$	$b = \cdot 0035$
$c = \cdot 0672$	$c = \cdot 0659$

so that the increase in the radius has reduced b to a nearly negligible quantity.

Logarithmic-law Condenser.*

A variable condenser with a different law is that used in the Kolster decremeter. In this it is required that the capacity should vary in accordance with the law of geometrical progression, *i.e.* if $c_1, c_2, c_3 \ldots$ be the capacities at equidistant points on the scale

$$\frac{c_2 - c_1}{c_1} = \frac{c_3 - c_2}{c_2} = \frac{c_4 - c_3}{c_3}, \text{ etc.}$$

For a rotary condenser, where θ is the displacement angle in degrees, the equation for the capacity is

$$c = ae^{m\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where a is the capacity for $\theta = 0$.

This capacity at 0 is provided by a small fixed condenser in parallel with the variable one.

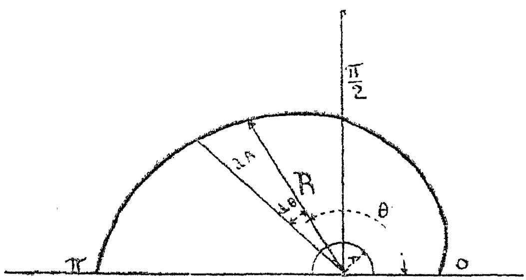


FIG. 10.

The equation for the radius from the centre to the edge of the moving vane is

$$R = \sqrt{2bme^{m\theta} + r^2} \quad . \quad . \quad . \quad . \quad (15)$$

where r is the radius of the circular space occupied by the spindle. etc., and b and m are constants depending on the minimum and maximum capacities of the condenser (see Fig. 10).

If the capacities at 180° and 0° be C_{180} and C_0

Then
$$\frac{C_{180}}{C_0} = k,$$

and if $C_0 = a$

$$C_{180} = ak, \quad \text{and } m = \frac{\log_e k}{180} \quad . \quad . \quad . \quad (16)$$

* Bull. Bur. Standards, vol. xi. p. 423.

Cylindrical Condenser.

A convenient form of variable condenser is that formed by one cylinder sliding over another.

If the distance between the cylinders be small compared with the radius, the capacity *per unit length* is

$$C = \frac{\frac{1}{2}K}{\log_e \frac{r_2}{r_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where K = dielectric constant of the dielectric between the cylinders, and r_1 and r_2 are the radii of the inner and outer cylinders.

CAPACITY OF RADIOTELEGRAPHIC ANTENNÆ.

The most complete set of formulas for calculating the capacity of radiotelegraphic antennæ of various forms are those given in papers by Prof. G. W. O. Howe, read before the British Association in 1914 and 1915.*

In the first paper Prof. Howe states that if a single straight aerial wire, either horizontal or vertical, be charged to a potential above or below that of the earth, then its electric charge will not be uniformly distributed over the surface of the wire, but will have a greater density near its ends. The calculation of the capacity is, however, greatly simplified if a uniform distribution be assumed. In this case the potential would vary from point to point in a manner which can be easily calculated.

A uniform distribution of charge is not possible in a continuous wire, but is possible if the wire be assumed to be made of a number of short pieces, each one centimetre long stuck end to end, but insulated from one another.

If when the distribution of potential is calculated the insulated pieces are connected together to form a continuous conductor, electricity will flow from the central parts to the ends, until all points are at a uniform potential which is approximately equal to the average potential of all the sections.

The capacity of a straight wire or other form of antenna depends not only on its dimensions, but on its distance from the earth.

Prof. Howe in his formulas first calculates the potential of the system considered as far removed from the earth, and then adds the extra potential due to proximity to the earth.

* *Wireless World*, Dec. 1914, Jan. 1915.

Straight Wire.

The formula for the average potential in absolute electrostatic units of a single straight wire (neglecting the earth effect) is

$$V_{av} = 4\pi r\sigma\left(\log_e \frac{l}{r} - \cdot 307\right). \quad . \quad . \quad . \quad (18)$$

where r = radius, and

l = length of the wire in centimetres,

σ = surface-density of the charge in electrostatic units per square centimetre.

The above is an approximate formula only, since it is incorrect in the immediate neighbourhood of the ends of the wire.

The accurate expression is

$$V = 4\pi r\sigma\left(\sinh^{-1}\frac{l}{r} - \sqrt{1 + \frac{r^2}{l^2}} + \frac{r}{l}\right) \quad . \quad . \quad (19)$$

The difference between the two formulas is negligible for all practical purposes, so that the first, which is simpler, may be used for all cases.

In Table 5, page 126, will be found values for the potential and capacity per metre of a single wire aerial.

Multiple Wire Antenna.

For a flat multiple-wire antenna

$$V_{av} = 4\pi r\sigma\left\{n\left(\log_e \frac{l}{d}\right) + \log_e \frac{d}{r} - B\right\} \quad . \quad . \quad (20)$$

where n = total number of wires in the antenna, and d is the distance in centimetres between them.

B is a factor depending on the total number of wires in the antenna.

Number of wires.	B.	Number of wires.	B.
2	0	8	6·40
3	·46	9	8·06
4	1·24	10	9·80
5	2·26	11	11·65
6	3·48	12	13·58
7	4·85		

The formula given above is not accurate when the length of the antenna is not a large multiple of its width.

For example, if $\frac{l}{d}=20$, and there are 12 wires, the ratio of length to width is only $\frac{20}{11}$, so that it is inaccurate to assume that $(\frac{11}{20})^2$ can be neglected when compared with 1, or that 1 can be neglected in comparison with $(\frac{20}{11})^2$.

The formula given above is obtained by finding the potentials at the middle points of each wire and taking their average value. This gives a potential in excess of the true average potential, and hence the capacity calculated from it will be too small.

The accurate expression is

$$V = 4\pi r\sigma\left\{n\left(\log_e \frac{l}{d} - \cdot 307\right) + \log_e \frac{d}{r} - B\right\} \quad . \quad (21)$$

which differs from the previous one by the term $\cdot 307$ only.

Prof. Howe has calculated the capacity of a number of antennæ by the application of the first formula, and finds that if the length be more than eight times the extreme width, the error is 3 per cent., and is 7·5 per cent. if the length be only twice the width, the true capacity being less than that calculated.

In Table 6 the capacity per metre of a number of parallel wire aerials will be found.

Four-Wire Aerial of the Box Type.

In this type the wires, in section, occupy the four corners of square.

If the side of the square be d, then for a uniformly distributed charge σ the potential of any wire due to its own charge will have an average value

$$4\pi r\sigma\left(\log_e \frac{l}{r} - \cdot 307\right)$$

due to the two nearest wires its average potential will be

$$2 \times 4\pi r\sigma\left(\sinh^{-1}\frac{l}{d} - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l}\right)$$

and due to the wire diagonally opposite

$$4\pi r\sigma\left(\sinh^{-1}\frac{l}{\sqrt{2}d} - \sqrt{1 + \frac{2d^2}{l^2}} + \frac{\sqrt{2}d}{l}\right)$$

Average potential of the wire and therefore of the whole aerial will be

$$V_{av} = 4\pi r\sigma \left\{ \log_e \frac{l}{r} - \cdot 307 + 2\left(\sinh^{-1}\frac{l}{d} - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right) \right.$$

$$\left. + \sinh^{-1}\frac{l}{\sqrt{2}d} - \sqrt{1 + \frac{2d^2}{l^2}} + \frac{\sqrt{2}d}{l} \right\}$$

$$= 4\pi r\sigma \left(\log_e \frac{l}{r} + \gamma \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

For all practical cases $\sqrt{1 + \frac{d^2}{l^2}}$ may be taken as unity with little error.

$\frac{l}{d} =$	20	50	100	150	200
$\gamma =$	7·58	10·22	12·26	13·48	14·33

The Increase of Capacity due to the Proximity of the Earth.

If the antenna be not far removed from the earth, its potential will be lowered, due to the induced negative charges on the earth, and the capacity will therefore be increased. The amount of this increase can be calculated easily with an accuracy greater than that required for all practical purposes.

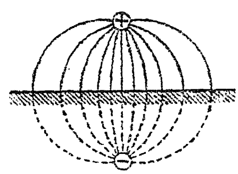


FIG. 11.

In most practical cases the ratio of the height to the length of the antenna is sufficiently large that it may be assumed that the negative charge induced in the ground is concentrated at a point in the centre of the electric image of the antenna. This image is situated at a distance below the surface of the earth equal to the actual height of the antenna above it.

In this case the potential due to the induced charge is

$$E = -\frac{Q}{2h}$$

where $\qquad Q = 2\pi r l\sigma$

so that $\qquad \mathbf{E} = 4\pi \mathrm{r}\sigma \times \frac{l}{4\mathrm{h}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$

Where the ratio of height to length is not so great, the formula No. (21) may be used, that has been obtained for the average potential of a wire due to a uniformly distributed charge on a parallel one, *i.e.*

$$E = 2\left(\sinh^{-1}\frac{l}{d} + \frac{d}{l} - \sqrt{1 + \frac{d^2}{l^2}} \right) \quad . \quad . \quad . \quad (24)$$

in which $d = 2h$, where h is the height of the antenna above the earth.

On page 127 will be found a table giving values of E for various ratios of $\frac{l}{2h}$.

Summary.

The formulas already found for the potential of a single wire and flat multiple wire aerial are corrected by including the above value for E. They become:

Potential of a single wire aerial

$$V_{av} = 4\pi r\sigma\left(\log_e \frac{l}{r} - \cdot 307 - \frac{E}{2}\right)$$
$$= 4\pi r\sigma\left(\log_e \frac{l}{r} - \cdot 307 - \frac{l}{4h}\right) \quad . \quad . \quad . \quad (25)$$

when $\frac{l}{2h}$ is less than 1.

For a flat-wire antenna

$$V = 4\pi r\sigma\left\{n\left(\log_e \frac{l}{d} - \cdot 307 - \frac{E}{2}\right) + \log_e \frac{d}{r} - B\right\} \quad . \quad (26)$$

For a four wire box antenna

$$\frac{Q}{2h} = \frac{2\pi rl\sigma \times 4}{2h} = 4\pi r\sigma \times \frac{l}{h} = 4\pi r\sigma \times 2E$$

$$V_{av} = 4\pi r\sigma\left(\log_e \frac{l}{r} + \gamma - \frac{l}{h}\right) \quad . \quad . \quad . \quad (27)$$

or

$$4\pi r\sigma\left(\log_e \frac{l}{r} + \gamma - 2E\right) \quad . \quad . \quad . \quad . \quad (28)$$

Potential of a Vertical Wire due to the Induced Charge on the Earth.

The potential at a point P on a uniform vertical wire due to the charge on the electric image of the wire is given by the formula

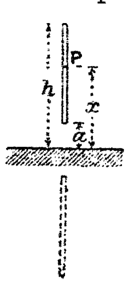


FIG. 12.

$$V_P = \log_e \frac{2(x+h)}{r} - \log_e \frac{2(x+a)}{r}$$

and for the average potential over the whole wire due to the image is

$$V_{av} = \frac{1}{h} \log 2^{2(a+h)} \left(\frac{a}{h}\right)^{2a}$$

$$= \frac{1}{h}\left\{2(a+h)\log_e 2 + 2a \log_e \left(\frac{a}{h}\right)\right\}. \quad (29)$$

when a is small compared with h, as it is in all practical cases.

The second term can be neglected as a is small, so that

$$V_{av} = 2 \log_e 2 = 1{\cdot}386 \times \text{charge per unit length.}$$

But for the cases most likely to arise in practice, where a is 3 or 4 feet from the ground, the potential is given by

$$V_{av} = \text{charge per unit length} \quad . \quad . \quad . \quad (30)$$

Two Wires meeting at Right Angles.

Consider the case of two wires AB and AC meeting at right angles, the wire AB having a uniformly distributed charge of one unit per centimetre.

Let l' be the length of AC the uncharged wire,
l ,, ,, AB the charged wire,
and let $m = \frac{l'}{l}$.

The average potential of AC due to the charge on AB is given by

$$V_{av} = \sinh^{-1}\frac{1}{m} + \frac{\sinh^{-1} m}{m} \quad . \quad . \quad . \quad . \quad (31)$$

The values of V_{av} for various values of m are given in Table 8.

Two Sets of Wires at Right Angles.

In this case, which is often met with in practice, to calculate potential accurately, it would be necessary to know the average potential of each uncharged wire (say the vertical ones) due to the charge on each horizontal wire.

Thus, for the case shown in Ex. II., p. 55, it would be necessary to work out the average potential of AC due to the charge on EF, and similarly for every other combination of pairs of wires.

This is, however, not necessary, as a sufficiently accurate determination can be made if it be assumed that all the n horizontal wires and the n vertical wires are bunched together into one horizontal and one vertical wire, separated by a distance equal to the mean distance between any one wire and the wires of the other set.

This mean distance for various numbers of wires is given by

n.	Mean distance.	n.	Mean distance.
2	·5 $\times d$	5	·386 $\times d$
3	·445 ,,	10	·366 ,,
4	·416 ,,		

The potential given by this method will not be quite correct. In Table 8 (b) (page 128) will be found curves showing the percentage

decrease in potential due to the fact that the two equivalent wires do not meet. The curves are for three ratios of $\frac{AA'}{l}$, and values for other ratios may be interpolated from them.

AA′ is the "mean distance" given above.
l is the length of the charged wire.

Two Wires inclined at an Angle.

For the average potential of the wire AB due to a uniformly distributed charge of one unit per centimetre on BC, where the angle ABC $= \gamma$.

Let $\beta = \cot\gamma$,

so that $$\frac{\beta}{\sqrt{\beta^2+1}} = \cos\gamma \qquad \frac{1}{\sqrt{\beta^2+1}} = \sin\gamma \qquad a' = m\cos\gamma$$

$$V_{av} = \sinh^{-1}\beta + \sinh^{-1}\left(\frac{1-a'}{a'}\beta\right)$$
$$+\frac{\cos\gamma}{a'}\left\{\sinh^{-1}\left(\frac{a'(1+\beta^2)}{\beta}-\beta\right)+\sinh^{-1}\beta\right\} \quad . \quad (32)$$

a' is given by DB $= a'l$ where D is the projection of AB on CB.

When the wires are equal in length

$$V_{av} = 2\{\sinh^{-1}\beta + \sinh^{-1}(\sqrt{1+\beta^2}-\beta)\}$$
or
$$= 2\log_e\{1+\sqrt{1+(\operatorname{cosec}\gamma+\cot\gamma)^2}\} \quad . \quad (33)$$

The values of V_{av} for various angles are given in Table 9.

The distance d has been calculated at which the wires would have to be placed parallel to each other so that the potential of each due to the charge on the other would be the same as it actually is with the wires at an angle.

This distance, as will be seen from the table, is always ·36 to ·37 of their greatest distance apart, so that fan-shaped antennæ and leading down wires can be treated as parallel wires with this value of d.

Potential of a Horizontal Wire of an Antenna due to the Image of the Vertical Wire or *vice versâ*.

There will be a certain decrease in potential of a horizontal aerial due to the fact that the capacity of the vertical part is increased by proximity to the earth, which has not been allowed for in the above calculations, and similarly for the influence of the horizontal part on the potential of the vertical wires.

This is allowed for by considering it as due to the image of the vertical part.

Since, however, the whole effect is small in comparison with the others which have been given above, it is usually sufficiently accurate to consider the whole charge of the image of the vertical wire to be concentrated at its mid-point, and to calculate the potential due to this charge for a distance equal to the mean distance of the horizontal wires from the concentrated charge from

$$V = \frac{\text{charge}}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

A more accurate value could be obtained by extending the calculations for two wires at right angles for the case where they do not meet, but it is not usually worth the extra work involved.

To obtain the capacity of a horizontal antenna with vertical leading down wires, the following potentials must be calculated :—

For the horizontal portion :

1. Potential due to its own charge (formula (18)).
2. ,, ,, the vertical wire (formula (31)).
3. ,, ,, its own image (formula (24)).
4. ,, ,, the vertical image (formula (34)).

For the vertical portion :

5. Potential due to its own charge (formula (18)).
6. ,, ,, the horizontal wire (formula (31)).
7. ,, ,, its own image (formula (30)).
8. ,, ,, the horizontal image (formula (34)).

The various tables referred to in the text may be used in many cases.

The following examples taken from Prof. Howe's paper will show how these calculations are made.

Example I.—Capacity of a single horizontal wire with vertical leading-down wire at centre, $l = 200$ feet, $h = 100$ feet, $r = \cdot 048$ inch, $\frac{l}{r} = 50{,}000$. Assume unit charge per centimetre length, then

$$2\pi r\sigma = 1$$

For the horizontal wire the potentials are :

1. Due to its own charge

$$2\left(\log_e \frac{l}{r} - \cdot 307\right) = 2(10\cdot 820 - \cdot 307) \quad = 21\cdot 03$$

2. Due to the vertical wire

(note $m = \frac{100}{100}$)

$$= \sinh^{-1}\frac{1}{1} + \frac{\sinh^{-1} 1}{1}$$

$$= 2 \sinh^{-1} 1$$

From Table 8 this is given as $= 1{\cdot}76$
or by reference to a table of hyperbolic functions it will be seen that $\sinh \theta = 1$ (nearly) for $\theta = {\cdot}88$.

3. Due to its own image

$\left(\frac{l}{d} = \frac{l}{2h} = \frac{200}{200}\right)$ $V_{av} = -{\cdot}94$ (Table 7)

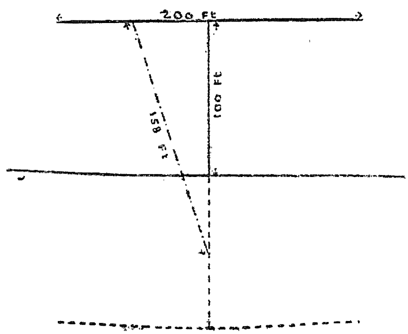


FIG. 13.

4. Due to image of vertical wire

$$\underset{\text{(approx.)}}{V_{av}} = \frac{\text{charge}}{\text{mean distance}} = \frac{-100 \times 30{\cdot}5}{y \times 30{\cdot}5}$$

(if lengths be kept in feet) $= \frac{-100}{158}$ (Fig.13) $= -{\cdot}63$

Total $= 21{\cdot}22$

Potential of the vertical wire:

5. Due to its own charge $= 19{\cdot}58$
6. Due to horizontal wire ($m = \frac{100}{100}$) $= 3{\cdot}52$
7. Due to its own image $= -1{\cdot}00$
8. Due to horizontal image $= -1{\cdot}27$

$20{\cdot}83$

The average potential of the whole antenna

$$= \frac{200 \times 21{\cdot}22 + 100 \times 20{\cdot}83}{300} = 21{\cdot}06$$

$$\left.\begin{array}{r}\text{Total charge on antenna}\\ \text{(at 1 unit per centimetre)}\end{array}\right\} = 300 \times 30{\cdot}5 = 9150 \text{ units}$$

$$\text{Capacity} = \frac{Q}{V} = \frac{9150}{21{\cdot}06} = 435 \text{ electrostatic units}$$
$$= 483 \text{ micromicrofarads}$$

EXAMPLE II.—Ten parallel wires 4 feet apart. $l = 600$ feet, $h = 200$ feet, $r = {\cdot}048$ in., with 10 leading-down wires at centre of aerial converging to a point near the earth.

$$\frac{l}{d} = 150\ ;\ \frac{d}{r} = 1000. \quad \text{Assume } 2\pi r\sigma = 1$$

Average potential of horizontal part:

1. Due to its own charge

$$V = \frac{Q}{C} = \frac{\text{length in cm.} \times \text{charge in mmfds. per cm.} \times \text{no. of wires}}{\text{total capacity}}$$
$$= \frac{600 \times 30{\cdot}5 \times \frac{10}{9} \times 10}{3{\cdot}845 \times 600} = \frac{33{\cdot}9 \times 10}{3{\cdot}845} \qquad = 88{\cdot}2$$

The capacity is taken from Table 6.

2. Due to vertical part

$$m = \tfrac{300}{200}\ ;\ \ n = 10$$

Average distance between vertical and horizontal wires in terms of the overall width

$$= {\cdot}366 \times 36 = 13{\cdot}2 \text{ feet (page 51)}$$

$$\frac{AA'}{l} = \frac{13{\cdot}2}{200} = {\cdot}066\ ; \text{ diminution per cent.} = 4{\cdot}3\ ^{*}$$
$$\therefore \text{ potential} = 14{\cdot}22 \text{ less } 4{\cdot}3 \text{ per cent.} = 13{\cdot}7$$

3. Due to its own image

$$\frac{l}{d} = \frac{l}{2h} = \frac{600}{400} \qquad = -13{\cdot}2$$

4. Due to vertical image
(mean distance 336 feet)

$$\frac{10 \times 200}{336} \qquad = -6{\cdot}0 \text{ approx.}$$

$$\text{Total} = \ \ 82{\cdot}7$$

* Estimated from Table 8 (*b*).

Average potential of vertical part :

5. Due to its own charge

Mean distance between converging wires

$4 \times \cdot 36$ feet $= 1 \cdot 44$ feet

$$\frac{l}{d} = \frac{200}{1 \cdot 44} = 1 \cdot 39 \; ; \quad \frac{d}{r} = \cdot 36 \times 1000 = 360$$

$\therefore$ capacity from Table (6) $= 4 \cdot 01$

$$\therefore \text{ potential} = \frac{33 \cdot 9 \times 10}{4 \cdot 01} \qquad = 84 \cdot 8$$

6. Due to horizontal position

$m = \frac{200}{300} =$ potential $= 2 \cdot 134 \times 10 \times 2$, for the two halves,

diminution 5 per cent. (Table 8 (*b*))

Potential $= 2 \cdot 134 \times 2 \times 10 \times \cdot 95 \qquad = 40 \cdot 4$

7. Due to its own image

$= 10$ times its charge per unit length $\qquad = -10$

8. Due to horizontal image

$$\frac{10 \times 600}{336} \qquad = 17 \cdot 9$$

Total $= 97 \cdot 3$

Hence we have 10×600 feet at average potential $82 \cdot 7$ and 10×200 feet at average potential of $97 \cdot 3$, or

10×800 feet at average potential of $86 \cdot 35$

$$\therefore C = \frac{800 \times 10 \times 30 \cdot 5}{86 \cdot 35}$$

$= 2825$ electrostatic units $= 3 \cdot 14$ millimicrofarads

Capacity of an Umbrella Antenna.*

The capacity of this form of antenna is found by calculating the potentials of the various parts in a similar manner to that adopted for horizontal aerials . . . . (35)

The following potentials must be calculated :—

For each rib :

(1) Potential due to its own charge (formula (18)).
(2) ,, ,, charges on the other ribs (formula (33)).
(3) ,, ,, charge on the vertical wire (formula (32)).
(4) ,, ,, image of the ribs (formula (34)).
(5) ,, ,, image of the vertical wire (formula (34)).

* *Wireless World*, Sept. 1915.

For the vertical wire:

(6) Potential due to its own charge (formula (18)).
(7) ,, ,, charges on ribs (formula (32)).
(8) ,, ,, its own image (formula (30)).
(9) ,, ,, image of the ribs (formula (34)).

The 1st and 6th terms are obtained from formula (18) already given.

The 4th, 5th, and 9th terms are obtained by the approximate method of formula (34).

The 8th term is obtained from formula (30).

The second term is obtained from formula (33).

To facilitate the calculation the angles between the various ribs of certain forms of aerials are given in the following table.

Umbrella aerials are usually made with the number of ribs given in the table, or simple multiples of them, and the angles for the latter case can easily be obtained from the table.

ANGLE BETWEEN RIBS FOR VARIOUS UMBRELLA AERIALS.

Angle with vertical.	$n=2$.	$n=3$.	$n=4$.		$n=5$.		$n=6$.		
α	θ	θ	θ_{12}	θ_{13}	θ_{12}	θ_{13}	θ_{12}	θ_{13}	θ_{14}
90	180	120	90	180	72	144	60	120	180
75	150	113·4	86·2	150	69·2	133·3	57·8	113·4	150
60	120	97·2	75·6	120	61	111	51·2	97·2	120
45	90	75·5	60	90	49	84·5	41·4	75·5	90
30	60	51·2	41·4	60	34·2	56·7	28·95	51·2	60

Table 10 gives the average potential of one rib due to the charges on all the others.

For the 3rd and 7th terms the formula No. (32) must be employed, since the ribs and vertical wires are usually of different lengths.

To simplify calculations the curves of Tables 11 and 12 are reproduced from Prof. Howe's paper, from which the following example is taken.

An aerial has six ribs each 30 metres long, and inclined at 66° with the vertical. The vertical wire is 26 metres long, and all wires 3 mm. in diameter.

Assume a uniform charge of unit quantity per centimetre, *i.e.* $2\pi r\sigma = 1$.

For a rib $\qquad \frac{l}{r} = \frac{3000}{\cdot 15} = 20000$

(1)	Potential due to its own charge		$= 19{\cdot}2$
(2)	,, ,, all the other ribs		$= 9{\cdot}35$
(3)	,, ,, the vertical wire $(m = \frac{30}{26})$		$= 1{\cdot}95$
(4)	,, ,, image of ribs charge $= -18000$ distance 40 metres		$= -4{\cdot}0$
(5)	,, due to image of vertical wire: charge $= -2600$ distance about 36 metres		$= -0{\cdot}7$
	Total potential of a rib		$= 25{\cdot}8$

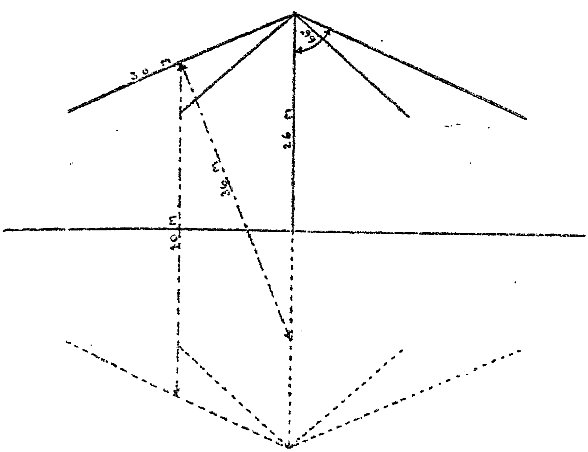


Fig. 14.

For the vertical wire $\frac{l}{r} = 17333$

(6)	Potential due to its own charge		$= 19{\cdot}0$
(7)	,, ,, the six ribs $(m = \frac{26}{30})$		$= 13{\cdot}5$
(8)	,, ,, its own image		$= -1{\cdot}0$
(9)	,, ,, image of the six ribs charge $= -18000$ distance about 36 metres		$= -5{\cdot}1$
	Total potential of vertical wire		$= 26{\cdot}4$

Hence there are $30 \times 6 = 180$ metres of wire at an average potential of 25·8, and 26 metres at an average potential of 26·4, making a total of 206 at an average potential of 25·8.

$$\text{Capacity} = \frac{20600}{25 \cdot 8} = 800 \text{ cms.} = \cdot 89 \text{ millimicrofarad}$$

Prof Howe gives a second example, in which each rib is composed of four wires. The capacity is found in the same manner as above, that of the ribs by means of formula (28).

Effect of Masts and Buildings.

In a recent paper * read before the British Association, 1916, Professor Howe shows the method by which the effect of masts and buildings, on the capacity of an aerial, may be calculated.

The effect of a cylindrical mast on the capacity of a single vertical wire of equal length placed parallel to it is obtained as follows :—

Assume the wire to have a uniform charge of one unit per centimetre. This will induce a negative charge of q units per centimetre on the mast. The images of the wire and mast in the earth will have charges equal and opposite to these values induced on them.

The value of q is determined by the fact that the resultant potential of the mast is zero. Having found q the potential of the wire and hence its capacity may be determined.

If l be the length of the wire or mast,

r the radius of the wire,

r_m „ „ mast and

D the distance between them,

the potential of the one due to the charge on the other is obtained from formula (24), *i.e.*—

$$V_{av} = 2\left(\sinh^{-1}\frac{l}{D} + \frac{D}{l} + \sqrt{1 + \frac{D^2}{l^2}}\right)$$

the values of which are given in Table 7.

The potential due to the charge on the conductor itself can be obtained from the same formula on substituting its radius r for D (see formula 19).

The rest of the calculation will be seen best from the following example given in Professor Howe's paper :—

* *Wireless World*, October and November, 1916.

EXAMPLE.—Let $l = 200$ feet $\qquad r = 05$ inches $= 1/240$ feet

$$r_m = \cdot 5 \text{ feet} \qquad D = 10 \text{ feet}$$

so that $\qquad \dfrac{l}{r} = 48{,}000 \qquad \dfrac{l}{r_m} = 400$

$$\frac{l}{D} = 20$$

The potential of the mast:

(1) Due to its own charge . . . . $= -11\cdot34q$
(from Table 7 or formula 19).

(2) Due to the charge on the wire . . . . . $= +\ 5\cdot48$
(ditto).

(3) Due to its own image . . . . $= +\ 1\cdot38q$
(formula 29).

(4) Due to the image of the wire . . . . . . $= -\ 1\cdot33$

The last term is obtained from the following consideration :—

If the wire and mast were both twice as long $\dfrac{l}{D}$ would be 40 and V_{av} would be 6·81 (Table 7).

This would be the average potential of the whole or of either half of the 400 ft. mast due to the 400 ft. wire; hence it is the potential of the actual 200 ft. mast due to the 400 ft. wire.

The potential of the actual mast due to the actual wire being 5·48, the difference, viz. 1·33, must be due to the charge on the image of the wire.

Since actual potential of the mast is zero,

therefore $\qquad 5\cdot48 - 1\cdot33 - (11\cdot34 - 1\cdot38)q = 0$

from which $q = 0\cdot417$ units per centimetre of length.

The potential of the wire is as follows :—

(5) Due to its own charge . . . . $= \ 20\cdot9$
(formula 18).

(6) Due to its own image . . . . $= -\ 1\cdot0$
(formula 30).

(7) Due to the charge on the mast $= (0\cdot417 \times 5\cdot48)$ $= -\ 2\cdot28$

(8) Due to the image of the mast $= (0\cdot417 \times 1\cdot33)$ $= +\ 0\cdot56$

Resultant potential . . . . $18\cdot2$

The presence of the mast lowers the potential of the wire from 19·9 to 18·2, and therefore increases its capacity 9·3 per cent.

Professor Howe shows that the potential of a wire running parallel to the wall of a building can be obtained in a similar manner by considering the image of the wire in the earth and the images of the wire and its earth image on the other side of the wall of the building.

A table showing a number of measured and calculated values for various arrangements of wires is given in the paper. The results agree within 4 per cent. except for one arrangement in which the wires were only 7·45 mm. apart.

CHAPTER III

THE MEASUREMENT OF INDUCTANCE

There are a large number of methods by which an inductance can be measured. In some the inductance is balanced in a bridge against resistances, condensers, or standard inductances. These methods are very suitable for air-core coils, of which the inductance is not dependent on the current strength, but for iron-core coils the inductance should be measured by a method in which the coil carries the full load current it is to work with.

The inductance is usually measured in henrys; in bridge methods, if resistances be in ohms and capacities in farads, the inductance is given in henrys; but if the capacity be in microfarads, the inductance will be in microhenrys.

The Fleming-Anderson Bridge.

This is perhaps the most useful bridge method for measuring an inductance. In addition to resistance coils, it only requires one standard condenser to set up the bridge, and may be used with alternating currents, direct current either with a make-and-break key or a buzzer interrupter.

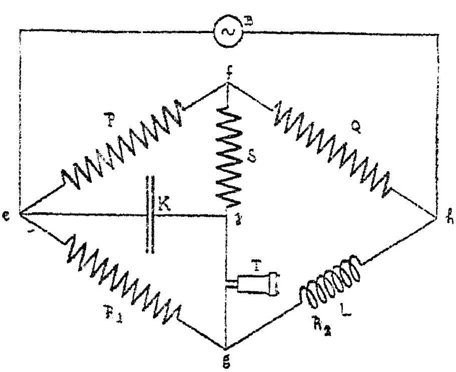


Fig. 15.

The diagrams of connections is given in Fig. 15.

The bridge is first balanced for direct currents, using an ordinary galvanometer, *i.e.*

$$\frac{P}{Q}=\frac{R_1}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and then, an alternating or intermittent direct voltage being applied to the points shown in the diagram, the resistance S is adjusted till the sound in the telephone is a minimum. A vibration galvanometer may be used in place of the telephone.

When this adjustment has been made

$$\mathbf{L} = \mathbf{K}\{\mathbf{S}(\mathbf{R}_1 + \mathbf{R}_2) + \mathbf{R}_1\mathbf{Q}\} \quad . \quad . \quad . \quad . \quad (2)$$

neglecting the power factor of the condenser (see page 84).

If K be in farads and the resistances in ohms, the inductance will be in henrys.

The resistance R_2 may be that of the inductance coil to be measured, or it may consist of this with a non-inductive resistance in series, which will considerably increase the range of inductance which can be measured by means of a given condenser and resistance box. By making the total resistance R_2 equal to some convenient round figure, for example, 100 or 1000 ohms, the calculations are simplified, which is a convenience where quick working is required.

Where great accuracy is required, or when measuring small inductances, the bridge should be balanced in the manner described, and then a second balance obtained with the inductance coil cut out, to give a zero reading by which the inductance of the bridge can be eliminated.

As the method is an important one, the proof is here given.*

Let the total quantities of electricity which have traversed the arms *ef*, *eg*, *ej*, from the time when the current was zero, be x, y, and z respectively.

Then the quantities of electricity traversing *eh* will be $x + z$, and for *gh* will be y, when the bridge is balanced so that no current traverses the telephone or other indicator.

The currents for the respective arms will be $\frac{dx}{dt}$ for *ef*, and so on; and the difference in potential between e and f for the arm P will be $P\frac{dx}{dt}$.

Along the arm *fje* the difference of potential will be

$$\frac{z}{K} + S\frac{dz}{dt}$$

so that

$$P\frac{dx}{dt} = \frac{z}{K} + S\frac{dz}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* Watson's *Practical Physics*, p 544.

Similarly, since j and g are at the same potential

$$\frac{z}{K} = R_1 \frac{dy}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The P.D. between g and h along the arm LR_2 is equal to

$$R_2 \frac{dy}{dt} + L \frac{d}{dt}\left(\frac{dy}{dt}\right) = R_2 \frac{dy}{dt} + L \frac{d^2y}{dt^2} \quad . \quad . \quad . \quad (5)$$

The P.D. between g and h along $gjeh$ is

$$S\frac{dz}{dt} + Q\frac{d}{dt}\left(x + z\right) = S\frac{dz}{dt} + Q\left(\frac{dx}{dt} + \frac{dz}{dt}\right) \quad . \quad . \quad (6)$$

Equating these last two quantities

$$S\frac{dz}{dt} + Q\left(\frac{dz}{dt} + \frac{dx}{dt}\right) = R_2 \frac{dy}{dt} + L \frac{d^2y}{dt^2}. \quad . \quad . \quad (7)$$

The values of $\frac{dx}{dt}$, $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ obtained from (3) and (4) are

$$\frac{dx}{dt} = \frac{1}{P}\left(\frac{z}{K} + S\frac{dz}{dt}\right)$$

$$\frac{dy}{dt} = \frac{z}{KR_1}$$

$$\frac{d^2y}{dt^2} = \frac{dz}{dt}\,\frac{1}{KR_1}$$

Substituting these values in equation (7)

$$\frac{dz}{dt}\left(S + Q + \frac{QS}{P} - \frac{L}{R_1K}\right) - \left(\frac{R_2}{R_1} - \frac{Q}{P}\right)\frac{z}{K} = 0 \quad . \quad (8)$$

but $\frac{R_2}{R_1} = \frac{Q}{P}$, so that the second term vanishes.

Hence $S + Q + \frac{QS}{P} - \frac{L}{R_1K} = 0$, from which

$$\frac{L}{R_1K} = S + Q + \frac{R_2}{R_1}S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or

$$L = K\{S(R_2 + R_1) + R_1Q\}$$

The Maxwell Bridge.

This bridge is shown in Fig. 16.

The conditions for balance are

$$\frac{P}{Q} = \frac{R}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\frac{L}{K} = RQ = SP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It has the disadvantage that the condenser must be continuously variable, or else a number of trial adjustments for the resistance values may be necessary, rendering the method tedious.

Butterworth's Bridge.

A modified bridge has been described by Mr. S. Butterworth,* by which a large range of inductances can be measured by means of a single condenser of fixed value (see Fig. 17).

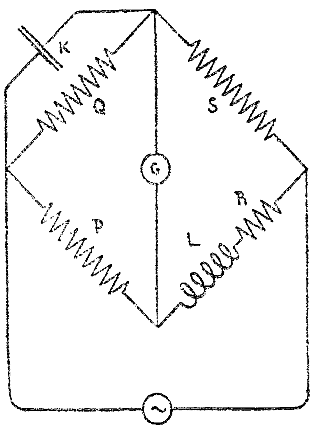


Fig. 16.

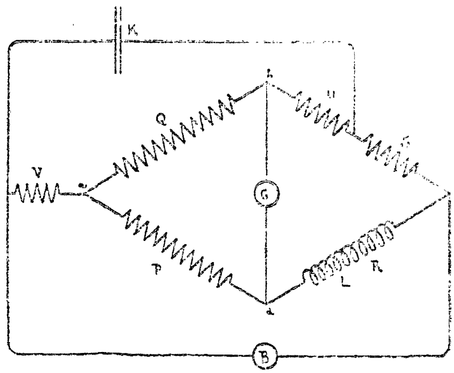


Fig. 17.

The conditions for balance are

$$RQ = P(U + S) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

$$\frac{L}{K}Q = S\{V(P + Q) + P(Q + U)\} \quad . \quad . \quad . \quad . \qquad (13)$$

so that by properly choosing S any inductance may be measured, and by adjusting V the inductive balance may be made independently of the resistance balance.

Owen's Bridge.

A very useful form of bridge is the one described by Mr. D. Owen.†

The connections are shown in Fig. 18.

* *Proc. Phys. Soc.*, xxiv. p. 210 (1912).

† *Ibid.*, xxvii. p. 38 (1914).

The two conditions to be fulfilled for a balance to be obtained are

$$\mathbf{K_3r_1 = K_4r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\mathbf{K_3r_1R_4 = K_4r_2R_4 = L} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The bridge is of such a form that fulfilling one of the above conditions will not tend to throw it out of balance for the other. Moreover, the balance is independent of the frequency, so that the applied voltage may be of any wave form. This allows a buzzer to be used if required.

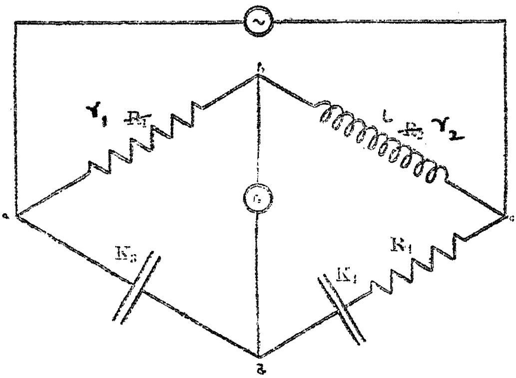


FIG. 18.

It will be noticed that the inductance L is proportional to R, so that any value of L can be measured if sufficient resistance be available.

Hay's Bridge.

The diagram for this is shown in Fig. 19.

The bridge is balanced by varying the resistance Q or the capacity K.

When balanced

$$L = \frac{PQK}{1 + K^2r^2\omega^2}; \quad R = \frac{PQK^2r\omega^2}{1 + K^2r^2\omega^2}$$

$$\frac{L}{R} = Kr\omega^2$$

where $\omega = 2\pi n$, $n =$ frequency.

For most practical cases $K^2r^2\omega^2$ is negligible compared with 1, so that the conditions reduce to

$$\mathbf{L = PQK; \quad R = PQK^2r\omega^2} \quad . \quad . \quad . \quad . \quad (16)$$

Inductance of Iron-Core Coils.

The inductance of an iron-core coil should be measured with the full current at the proper frequency passing through it.

For this purpose bridge methods are unsuitable in most cases, except where the total current is small, *e.g.* in measuring the inductance of a telephone receiver.

If a sinusoidal current i amperes be passed through a coil of inductance L (henrys), and the voltage at the ends of the coil be e,

then $$i = \frac{e}{\sqrt{R^2 + (2\pi nL)^2}} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

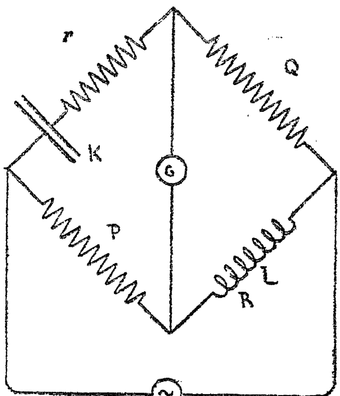


Fig. 19.

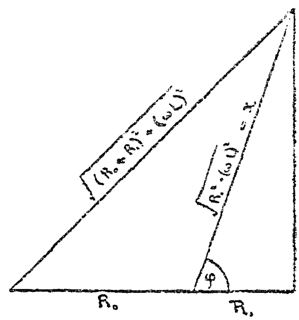


Fig. 20.

where R is the ohmic resistance of the coil, and n is the frequency, and there will be a difference in phase between e and i given by

$$\tan \phi = \frac{2\pi nL}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

An inductance can therefore be measured if e and i be measured, and either R or ϕ be measured, the frequency being known.

For many inductances the resistance is negligible as compared with the inductance, so that approximately

$$\frac{e}{i} = 2\pi nL \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

A more exact value can be found by measuring R either on the Wheatstone bridge or by noting the volt-drop when a measured continuous current is sent through the coil, and inserting its value in equation (17).

The Drysdale Alternating-Current Potentiometer.

The method described above is not suitable for measuring either very large or very small inductances, since these necessitate the use of ammeters and voltmeters whose range is outside the normal limits for every-day instruments. In these cases especially, and also for all other inductances (and many other measurements), the Drysdale alternating-current potentiometer is very useful.

The instrument consists of an ordinary potentiometer such as is used for continuous measurements. In place of the accumulator used to supply the current in the resistance coils, a special form of transformer is used, the power for which is supplied from the same generator as that used for the inductance circuit.

To maintain the current in the potentiometer at a constant value, a dynamometer milliammeter is included in the circuit.

The milliammeter can be checked if necessary by using the potentiometer with continuous current (for which purpose it is adapted), and checking the readings given by the instrument when the potentiometer current is set at its proper value by means of a standard Weston cadmium cell in the usual manner.

The indicating instrument is usually a Tinsley vibration galvanometer for low frequencies, and a Duddell vibration galvanometer for higher frequencies. These instruments are described in Chapter VI. A telephone receiver may be substituted in many cases.

In order to obtain a null deflection in the galvanometer, the volt-drop in the potentiometer coils must be equal to that in the circuit being measured, and these two potential differences must be in phase.

The equalisation of the volt-drop is carried out by adjusting the resistance in the potentiometer circuits exactly as for continuous-current work, and the bringing of the two potentials into phase is effected by the special transformer by which the potentiometer is supplied with current.

This transformer consists of two fixed primary windings and a secondary winding, which can be rotated on an axis with reference to the two primary coils.

These primary coils are wound at right angles, and when supplied with two-phase current produce a rotating field at the centre, where the secondary coil is placed. By rotating this secondary, the difference of phase between the current in the potentiometer and one phase of the supply can be made to have any value.

In place of using a two-phase supply, a single-phase supply can be used, a condenser and resistance being connected so as to give the current of the required difference of phase from the supply, for the second circuit of the transformer. This is the more usual procedure.

A diagrammatic sketch of the transformer and its connections is given in Fig. 21.

The method of measuring an inductance with this instrument is as follows.

A non-inductive resistance capable of carrying the current to be used in making the measurement is connected in series with the inductance and any other apparatus required.

The volt-drop for this resistance is measured, and the reading of the dial on the phase-shifting transformer is noted. A movable pointer is provided, which can be set to zero, so that all phase angles are measured directly.

The volt-drop for the inductance is now measured, a volt box being used if necessary.

When a balance is obtained the pointer of the phase-shifting transformer indicates the difference in phase between the *pds* for the resistance and the inductance.

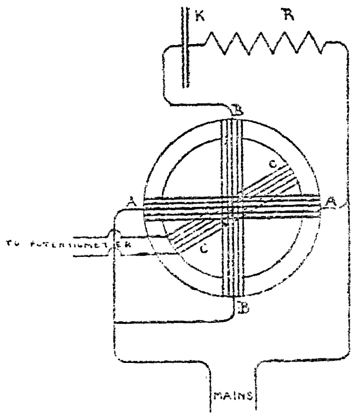


FIG. 21.

The inductance is obtained from

$$2\pi nL = \frac{e}{i}\sin\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and the resistance of the coil from

$$R = \frac{e}{i}\cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$\phi = \text{phase angle}$$

The current i is given by

$$i = \frac{e_0}{r_0}$$

where e_0 is the volt-drop for the non-inductive resistance r_0.

By measuring the volt-drop for the non-inductive resistance and inductance in series, additional observations can be obtained to check the other figures by drawing the vector diagram and noting whether the vectors form a closed figure (see Fig. 20).

Measurement of Self-Inductance by the Campbell Mutual Inductometer.

Mr. A. Campbell, in his paper on the mutual inductometer (see page 72), shows that this instrument affords a useful method for measuring small self-inductances.

The connections are shown in Fig. 22, where coil a is the fixed coil of the inductometer, and b a coil to be measured. The conditions of balance are

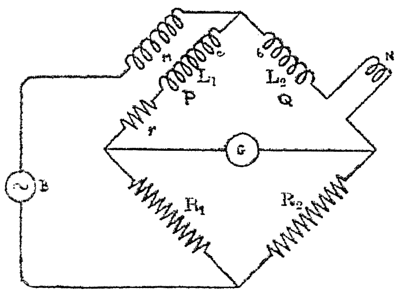


FIG. 22.

$$R_2P = QR_1$$

$$R_2(L_1 + M) = R_1(L_2 - M) \quad (22)$$

P being the total resistance of the arm.

Where the non-inductive arms R_1, R_2 are equal, we have

$$L_2 - L_1 = 2M \quad (23)$$

so that if $L_1 = L_2$ the inductometer balances at $M = 0$.

If now a small self-inductance N be inserted in the bridge in series with L_2 and a variable non-inductive resistance r in series with L_1, the inductance of N can be measured directly.

The bridge is balanced with N cut out of circuit, which gives a reading of $M = 0$, or a small value due to the inductance of the rest of the circuit.

N is now put in circuit, and r and M varied for a new balance, when

$$N = 2M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and r gives the resistance of N.

The most sensitive arrangement is when a and b are the two halves of the fixed coils of the mutual inductometer. The value of this method is that the residual inductance of the leads can be measured more easily than by methods involving self-inductances only, since the inductometer will measure from a positive to a small negative value through zero.

Mutual Inductance.

The mutual inductance between two circuits can be measured by any method by which self-inductance can be measured.

If the two coils, for which the mutual inductance is required, be connected in series, so that their fields are in the same direction, the total inductance of the combination is

$$L_1 + L_2 + 2M \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

By reversing the connections of one coil the total inductance is

$$L_1 + L_2 - 2M \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where L_1 and L_2 are the self-inductances of the respective coils, and M is the mutual inductance between them.

The difference between the above values is 4M.

The principal drawback to this method is that in many cases the bridge used must be capable of measuring a very much larger inductance than the value of the mutual inductance required. This is the case when L_1 and L_2 are widely different, even when they are tightly coupled.

Maxwell's Bridge.

Maxwell has given a bridge method by which the ratio of the mutual inductance between two coils to the self-inductance of one of them can be measured.

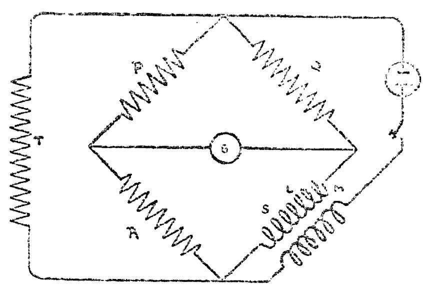


FIG. 23.

The connections are shown in Fig. 23. The bridge is first balanced for steady currents, and then the value of T is adjusted till there is no throw of the galvanometer on breaking or making the galvanometer circuit.

The conditions for balance are

$$PR = QS$$
$$L/M = \{1 + Q/P + (Q + S)/T\} \quad . \quad . \quad . \quad (27)$$

This method is applicable only to the case where the mutual inductance between the coils is less than about half the self-inductance of the coil in the bridge arm.

Lees' Bridge.*

Prof. C. H. Lees has recently described a method by which the mutual inductance can be obtained, whether it is large or small compared with L.

The connections are shown in Fig. 24.

The balance is not a continuous one for all changes of E.M.F., but is an aggregate or integral balance from one steady state to another.

The procedure to facilitate the measurement is as follows.

After the steady current balance has been made, the direction of the ballistic throw for the self-inductance of L is noted, the mutual inductance being cut out of action by opening the circuit of resistance U.

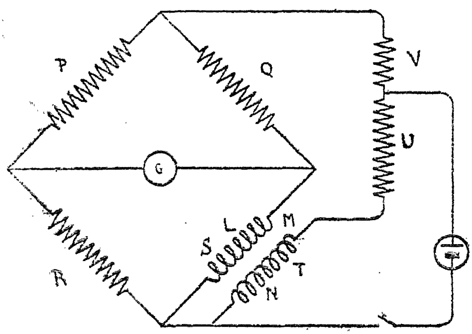


FIG. 24.

Connect the mutual inductance in circuit, and note the galvanometer throw. If it is larger than the self-inductance throw, the connections of the coil N should be reversed.

If now the throw is of opposite sign to the self-inductance throw, increase U till the throw is reduced to zero, V being zero.

If the throw be in the same direction as the self-inductance throw, make U zero and adjust V for zero throw.

The arrangement with V zero was first described by Brillouin.

The conditions of balance are

$$P/Q = R/S$$
$$L/M = \{Q + S + (1 + Q/P)V\}/(T + U) \quad . \quad . \quad (28)$$

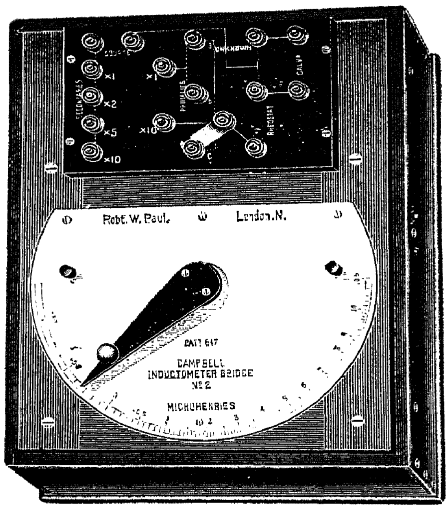
S = resistance of coil with inductance L,
T = ,, ,, ,, N.

A mutual inductance can be measured if a variable standard of mutual inductance be available.

The Campbell Inductometer.†

Mr. A. Campbell has designed a useful type of variable mutual

* *Proc. Phys. Soc.*, xxvii. p. 89 (1916).
† *Proc. Phys. Soc.*, vol. xxi. p. 75, April, 1908, vol. xxii. p. 207, 1910.



CAMPBELL INDUCTOMETER BRIDGE.

(*Page* 72.)

inductance, and has given a number of applications of its use in the measurement of mutual and self-inductances.

The general scheme of the construction of this is shown in Fig. 25.

The primary circuit consists of two equal coaxial coils, c and c', which are connected in series so that their fields are in the same direction.

The secondary circuit consists of two coils, D and F, in series, of which D is movable, being mounted on an axis at a, which is eccentric with respect to the axis of c and c', the rotation being measured by a pointer fixed to D.

By adopting this form of construction the scale of mutual inductance for various positions of D is a convenient one, and, in particular, includes a point for which the mutual inductance is zero and a short travel on the negative side.

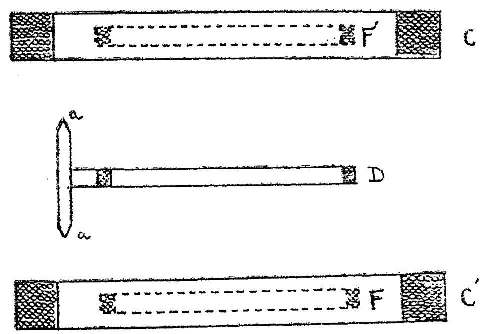


FIG. 25.

The coil F is wound with a cable of ten separately insulated strands, so that the mutual inductance of each of them with the primary is the same, the value being made equal to the maximum value of the mutual inductance for the coil D.

Thus in one pattern the mutual inductance between D and the primary coils varies from 0 to 10 microhenrys, and each of the coils of F has a mutual inductance of 10 microhenrys with the primary coils.

The coils of F can be connected in series by means of a switch, so that the mutual inductance is the sum of that due to the coils of F in circuit (with that due to the moving coil).

A second fixed coil similar to F, but for which each coil has 10 times the mutual inductance of those of F, is provided, so that the total range in this pattern is 1110 microhenrys.

To measure a mutual inductance with this apparatus when the

value is less than that of the variable standard, the connections of Fig. 26 are used, where

A = primary of the standard,
C = secondary " "
B = primary of the unknown,
D = secondary " "

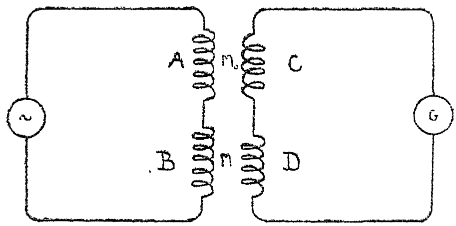


FIG. 26.

The coils C and D must be connected in opposition, which is soon ascertained by trial.

The balance is indicated by silence in the telephone, and

$$M = M_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where M_0 is the scale reading of the standard.

When the mutual inductance is greater than that of the standard, the following method should be used.

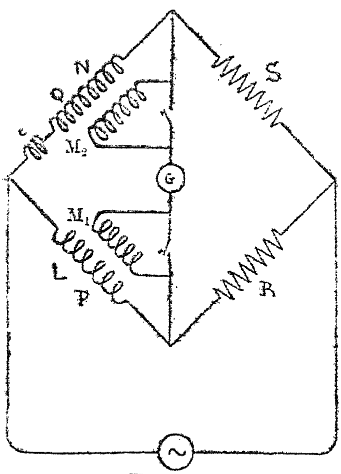


FIG. 27.

Connect the primary coils of the standard as shown in Fig. 27, adding, if necessary, to the latter a coil, c, which will make the total self-inductance N of this arm greater than that of the standard.

The conditions of balance are

$$\left.\begin{aligned} PS &= RQ \\ NS &= LR \end{aligned}\right\} \quad . \quad (30)$$

so that the coil c should preferably be one whose inductance is continuously variable, so that the inductance balance can be made without disturbing the adjustment of the resistance balance.

Having obtained this balance, the secondaries are connected in

series (in opposition) with the telephone or galvanometer, and the variable mutual inductance is adjusted till a balance is indicated, the conditions of balance being

$$RM_2 = SM_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

M_1 being mutual inductance of coil D.

If a variable self-inductance is not available, a non-inductive resistance box must be connected in series with one of the inductance arms, and it should include a slide wire or other continuously adjustable resistance.

Measurement of Mutual Inductance by the Ballistic Galvanometer.

A ballistic galvanometer may be used for measuring a mutual inductance, although the experiment usually takes the form of calibrating the galvanometer by means of a mutual inductance.

When a quantity of electricity Q is discharged through a ballistic galvanometer, the formula connecting Q with the deflection is

$$Q = \frac{T}{\pi} k \sin \frac{a}{2}\left(1 + \frac{\lambda}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where the symbols have the same meaning as on page 86, Chapter IV.

If the secondary of a mutual inductance be connected in series with the galvanometer, and a current i sent through the primary, then when the primary current is broken the quantity of electricity discharged through the galvanometer is $\frac{Mi}{R}$, where R is the total resistance in the galvanometer circuit, and Mi and R are all in electromagnetic units.

Hence

$$\frac{Mi}{R} = \frac{T}{\pi} k \sin \frac{a}{2}\left(1 + \frac{\lambda}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Mutual Inductance by the Grassot Fluxmeter.

In place of using an ordinary ballistic galvanometer, the Grassot fluxmeter affords a convenient method for measuring a mutual inductance by this method.

The fluxmeter is a moving-coil galvanometer specially designed so that it is quite dead-beat, *i.e.* the pointer takes up its final reading in the first deflection, without any swinging, and remains stationary until the circuit is again disturbed. Hence observations are made with this instrument with much greater ease than with an ordinary galvanometer.

This property of the fluxmeter is due to its special design. In moving-coil galvanometers the control by which the deflection of the coil is limited is due to three causes :—

(1) The controlling force due to stiffness of the suspension, which is proportional to the angle turned through by the coil.

(2) Damping forces due to the friction of the air, which are roughly proportional to the angular velocity of the coil.

(3) Electromagnetic damping forces due to the currents reduced in the coil by its movement in the magnetic field, which are proportional to the angular velocity of the coil.

In an ordinary ballistic galvanometer the first is the predominant control, whilst (2) and (3) are small. In the fluxmeter, however, (1) and (2) are small, and practically the whole control is due to (3); the instrument being practically free from any control tending to bring the needle back to its initial position.

In consequence of this the final deflection does not (within limits) depend on the rate at which the change of flux through the secondary circuit takes place.

The inductance is given by

$$M = \frac{5 \times \text{change of flux}}{C} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where the change of flux is obtained directly from the scale of the instrument.

M is given in centimetres if C be in amperes.

The Drysdale Combined Inductance and Capacity Testing Bridge.

Dr. C. V. Drysdale has devised a very convenient instrument by which the measurement of inductance and capacity by the various bridge measurements enumerated here and in Chapter IV. may be easily carried out.

It consists of four sets each of four resistance coils of 1, 10, 100, and 1000 ohms respectively, connected so that any other instrument, such as an inductance or condenser, can be placed in series or in shunt to any of the four arms.

Terminals for connecting the current supply and detector are provided, these being symmetrically arranged and placed close together, so that twin connecting leads may be used to eliminate inductive disturbance.

The diagram of connections is shown in Fig. 28.

As an illustration of the use of this bridge, Fig. 29 shows the connections for Maxwell's test of inductance with a variable standard condenser.

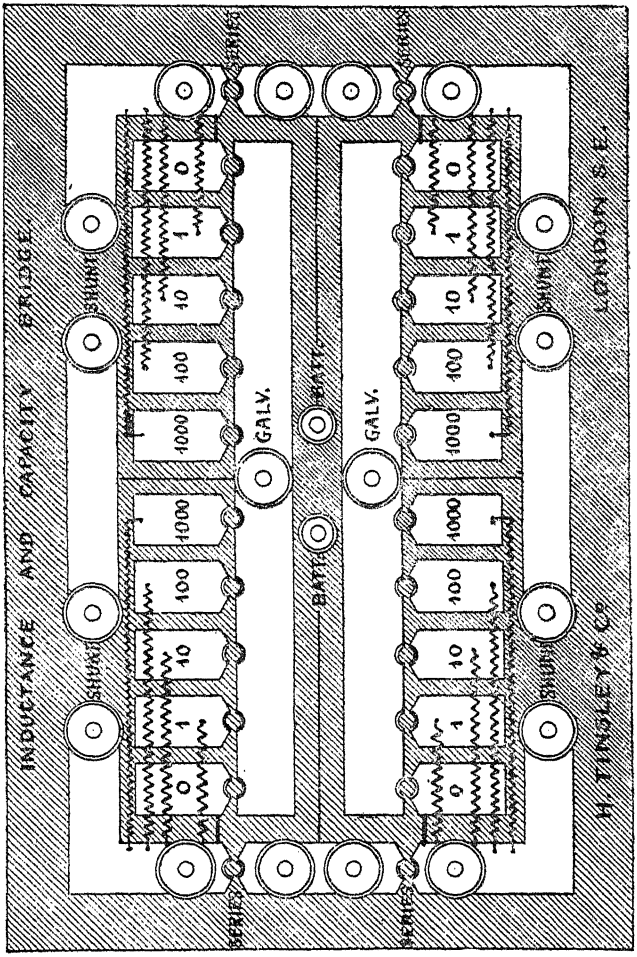


Fig. 28.

The coil under test (L) is connected in one of the series gaps, and a variable condenser K in the opposite shunt gap.

Select suitable resistances p, q, r, s in the four arms, such that

$$ps = qr$$

and connect a resistance box R in the shunt gap next L.

Vary K and R till balance is obtained, when

$$L \text{ (henrys)} = Kps \div 10^6 \quad \text{(K in microfarads)}$$

resistance of coil tested

$$= \frac{r^2}{R + r}$$

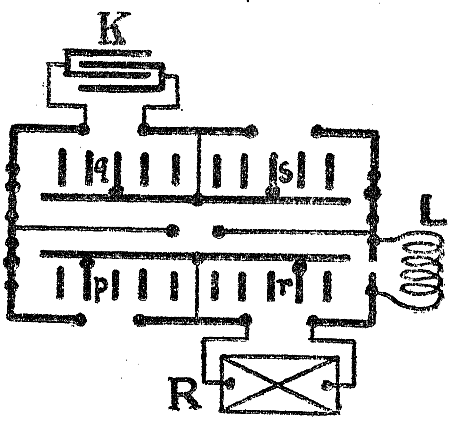


Fig. 29.

The connections for a large number of other tests will be found in a pamphlet issued by the makers, Messrs. H. Tinsley and Co.

Carey Foster Bridge.

This bridge, which is described on page 84, may be used for measuring mutual inductances.

CHAPTER IV

THE MEASUREMENT OF CAPACITY

To measure the capacity of a condenser there are two main principles on which the various methods are based.

In the first place, the condenser can be charged by means of a steady potential V, and its capacity determined from the relationship

$$C=\frac{Q}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where Q is the total quantity of electricity stored in the condenser by the charge.

In the second place, the condenser can be placed in an alternating-current circuit, and, by measuring the impedance, the capacity determined from

$$\text{Impedance}=\frac{1}{pC} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $p=2\pi \times$ frequency.

For a perfect condenser, that is, one in which there is no source of loss of energy, either method will give the true capacity, that is, the value which may be calculated from the geometric dimensions of the condenser and the dielectric constant of the material of which it is made.

The majority of condensers depart to a greater or less extent from this perfect condition. Thus when tested with continuous current, part of the charge is "absorbed" by the dielectric and does not flow out of the condenser immediately on discharge. If, after discharge, for a short time, the condenser terminals be insulated, on testing, after a short period of rest, it will be found that a futher quantity of electricity can be discharged from the condenser.

When tested with alternating current, for a perfect condenser, the phase of the current is 90° ahead of the impressed E.M.F., and there is no loss of energy in the condenser, but for most

condensers there is a loss, and hence there is an energy-component of the current in phase with the impressed E.M.F.

If the actual angle of lead of the current ahead of the impressed E.M.F. be ϕ, then the power-factor of the condenser is $\cos\phi$, and if θ be the angle $\left(\frac{\pi}{2}-\phi\right)$ we have

$$\cos\phi = \sin\theta$$

and θ may be termed the phase-difference of the condenser.

For well-insulated air-condensers, of which the resistance of the leads and connections between the plates is small, the angle ϕ is sensibly equal to 90° or $\frac{\pi}{2}$, and therefore

$$\cos\phi = \sin\theta = 0$$

but for other condensers such as those made with waxed paper, ebonite, glass and mica, ϕ will differ from $\frac{\pi}{2}$ by a greater or less degree.

From the vector diagram (Fig. 30), which represents the vectors

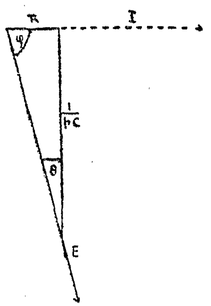


FIG. 30.

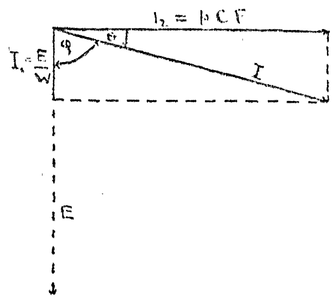


FIG. 31.

of an imperfect condenser, it may be seen that it is equivalent to a capacity C in series with a resistance r, where

$$\tan\theta = pCr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$p = 2\pi$ times the frequency.

Fig. 31 is a second vector diagram for the current in the condenser. In this case the condenser is equivalent to a capacity C in parallel with a resistance W, where

$$\tan\theta = \frac{1}{pCW} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For most condensers θ is a small angle, so that r is a relatively small and W a relatively large resistance.

A knowledge of the power factor is essential for many purposes in which the condenser may be employed as a measure of capacity, and gives reliable information as to the order of magnitude of the absorption effects.

The power factor can be determined by measuring the angle θ directly, or by measuring the energy loss of the condenser and computing θ.

ALTERNATING CURRENT METHODS.

When using alternating current in the measurement of capacity, it is essential that a supply with a pure sine wave form should be used, since in these tests the quantity measured is the impedance of the condenser, or

$$\frac{1}{2\pi n C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

If the wave form is not a sine curve, it is equivalent to a number of sinusoidal curves of different frequencies (these frequencies being harmonics of the fundamental) superimposed, the amplitudes being different.

Hence the quantity measured would be

$$\Sigma \frac{1}{2\pi n C} k'$$

where n will have all the values of frequency into which the wave form can be analysed, and k' is a factor depending on the amplitudes.

Even if it were easy to analyse the wave form into its components, it would be laborious to work out the results of a measurement.

A pure sine wave can be obtained by interposing a wave filter between the supply and the test apparatus. A description of some forms of wave filters is given in Chapter VI.

Wien's Series Resistance Bridge.

The following method, due to M. Wien, is perhaps the most convenient method for measuring the capacity and power factor of a condenser using alternating current.

The connections are shown in Fig. 32, in which R_3 and R_4 are non-inductive resistances, C_1 the condenser to be measured, and C_2

a condenser of which the power factor is practically zero or is known.

The conditions of balance are

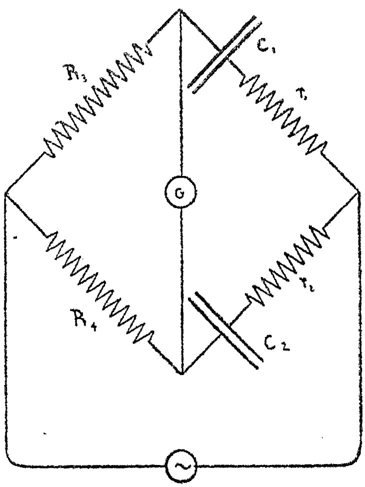


FIG. 32.

$$\left.\begin{aligned} \frac{C_2}{C_1} &= \frac{R_3}{R_4} \\ \frac{r_2}{r_1} &= \frac{R_3}{R_4} \end{aligned}\right\} \quad . \quad . \quad (6)$$

where r_1 is the equivalent series resistance of condenser C_1 (including any resistance actually put in series with it), and r_2 is the same quantity for C_2.

For the power factor

$$pC_2r_2 = pC_1r_1 \quad . \quad (7)$$

where $p = 2\pi n$, n being the frequency. If the series resistances r_1 and r_2 be made up of ρ_1 and q_1 and ρ_2 and q_2 respectively, where ρ_1 represents the equivalent resistance of the condenser and q_1 an added series ohmic resistance, the above becomes

$$pC_2(\rho_2 + q_2) = pC_1(\rho_1 + q_1) \quad . \quad . \quad . \quad . \quad (8)$$

which may be written

$$\tan \theta_2 - \tan \theta_1 = pC_1r_1 - pC_2r_2 \quad . \quad . \quad . \quad (9)$$

which for the small angles usually met with reduces to

$$\tan (\theta_2 - \theta_1) = pC_1r_1 - pC_2r_2 \quad . \quad . \quad . \quad (10)$$

This is a useful method when it is not known which condenser has the smaller power factor.

Fleming and Dyke's Bridge.

In carrying out tests on power factor and conductivity of dielectrics, Fleming and Dyke made use of the bridge shown in Fig. 33.

The condensers C_3, C_4 actually used were variable, but where the object is to measure a capacity only, they might with advantage be of fixed value. C_1 is the condenser whose capacity and power factor are required, and C_2 a variable condenser with a non-inductive resistance in series.

The conditions of balance are

$$\frac{C_3}{C_4} = \frac{C_1}{C_2} + R_2 S \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\frac{S}{C_1 p} = pC_2R_2 = \text{power factor} \quad . \quad . \quad . \quad (12)$$

where the symbols have the same meaning as in the previous method, but S is the equivalent parallel resistance of the condenser C_1.

Series Inductance Method.*

In this method the two condensers are balanced against resist-

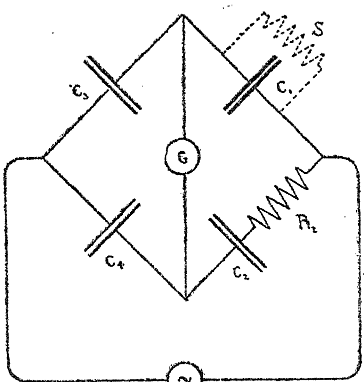


Fig. 33.

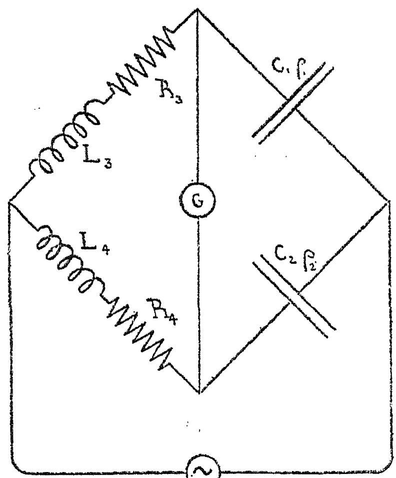


Fig. 34.

ances, but an inductance is connected in series with each resistance, the connections being as shown in Fig. 34.

Only one inductance is necessary, but by using two, one in each arm, the use of very small inductances is avoided, and it is not necessary to know beforehand which condenser has the larger power factor. Moreover, the correction for the inductance of the leads, etc., is simplified.

One inductance must be continuously variable, whilst the other may be of fixed value.

This method has the advantage that the device for compensating the losses of the condenser is not included in the same arm, and thus

* Grover, "Capacity and Power Factor of Condensers," Bull. Bur. Standards, vol. 3, No. 3. Reprint No. 64.

will not alter the capacity of that arm, an important point where small capacities are being tested.

The ratio of the capacities is given by

$$\frac{C_1}{C_2}=\frac{R_4}{R_3}\left(1+\frac{pL_3}{R_3}\tan\theta_2-\frac{pL_4}{R_4}\tan\theta_1\right)$$

$$=\frac{R_4}{R_3}(1+\tan\phi_3\tan\theta_2-\tan\phi_4\tan\theta_1)$$

where $$\frac{pL_3}{R_3}=\tan\phi_3, \text{ etc.} \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

The correction term in the brackets is in most cases small.

The Wien series resistance bridge, the parallel resistance bridge, and the series inductance bridge, can be used for comparing two condensers simultaneously, but Dr. Grover * shows that more accurate results will be obtained by using one condenser as a standard and successively substituting the other condensers to be compared, in the fourth arm of the bridge.

The Fleming-Anderson Bridge.

This bridge may be used to measure a capacity exactly as when measuring an inductance. The diagram of connections is given in Fig. 15, and a description on page 62, Chapter III.

If the condenser be one whose power factor is not negligible, the conditions for balance are modified as follows :—

$$L=KR_1\left[\frac{S(P+Q)}{P}+Q\right](1-\tan_2\theta) \quad . \quad . \qquad (14)$$

$$R_2=\frac{Q}{P}R_1+p^2LK\rho$$

$$=\frac{QR_1}{P}+pL\tan\theta \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

when $P=Q$, $$L=KR_1(2S+Q)(1-\tan^2\theta) \quad . \quad . \quad . \quad . \qquad (16)$$

$$R_2=R_1+pL\tan\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

Carey Foster Bridge.

This method involves the use of a mutual inductance, which, for convenience of work, should be a variable one.

The diagram of connections is shown in Fig. 35.

* Simultaneous measurement of the capacity and power factor of condensers. (Bull. Bur. Standards, vol. 3, No. 3. Reprint No. 64.)

The conditions for balance are

$$C = \frac{M}{RP} \quad . \quad . \quad . \quad (18)$$

$$S = R\frac{L - M}{M} \quad . \quad (19)$$

where S includes the equivalent series resistance of the condenser.

For convenience of work the total resistance of the P arm (which includes the resistance of the inductometer coils) is an even hundred or thousand ohms, and similarly for the R arm. Special auxiliary boxes for this are provided with the Campbell inductometer.

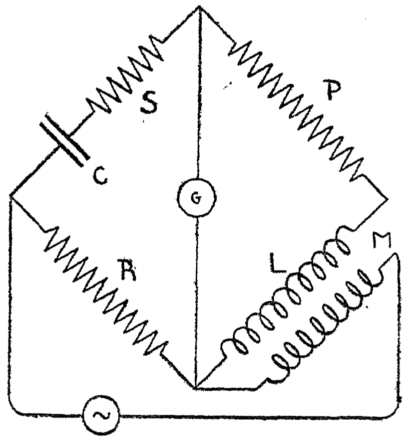


FIG. 35.

DIRECT-CURRENT METHODS.

When a condenser made with solid dielectric is charged with direct current, the quantity of electricity which enters the condenser consists not only of the quantity given by $Q_1 = CV$, but a further quantity which is absorbed by the dielectric, and the magnitude of which depends on the time the charging voltage has been applied before discharge.

On discharging the quantity Q_1 is discharged, and part of the absorbed quantity will leak out also, the amount depending on the time during which the discharge takes place.

If the condenser be insulated for a time, part of the absorbed quantity will become free and a further quantity can be discharged.

Hence in measuring the capacity of a condenser which shows absorption, the conditions as to time of charge and discharge should be specified.

The Ballistic Galvanometer.

In this method the condenser is charged to a known voltage, and the quantity of electricity measured by discharging through the galvanometer (see Fig. 36).

The usual method is to compare the throws given by the condenser and a standard whose capacity is known.

The throw of the ballistic galvanometer is proportional to the quantity of electricity discharged through it, provided the throws

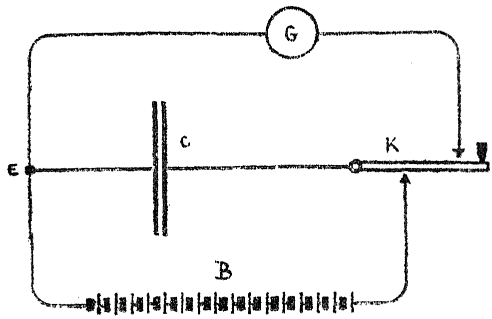


FIG. 36.

are not too great. Hence the capacities are in the same ratio as the throws.

The capacity can, however, be determined absolutely by this method, *i.e.* without reference to another condenser.

The formula becomes

$$C = \frac{Q}{E} = \frac{T}{\pi}\frac{k}{E}\sin\frac{a}{2}\left(1 + \frac{\lambda}{2}\right) \quad . \quad . \quad . \quad . \quad (20)$$

where T is the time of a complete vibration of the galvanometer suspension in seconds,

k is the reduction factor of the galvanometer,

a is the amplitude of the first swing or throw of the galvanometer,

λ is the logarithmic decrement of the vibration of the galvanometer,

E is the charging potential in volts,

C is the capacity in farads.

The logarithmic decrement λ may be measured by noting the ratio of the amplitude of vibration of successive swings of the galvanometer when allowed to vibrate freely.

If D_1 be the amplitude of the first, and D_n that of the nth vibration

$$\lambda = \frac{1}{n-1}\log_\epsilon\left(\frac{D_1}{D_n}\right) \quad . \quad . \quad . \quad . \quad (21)$$

The reduction factor k is given by $k = \frac{\mathrm{H}}{\mathrm{G}}$, where H is the strength of the magnetic field in which the suspension is placed, and G is a constant depending on the dimensions of the coils of the galvanometer.

k is eliminated by observing the steady deflection produced by a known current A.

For a moving magnet galvanometer

$$k \tan \theta = \mathrm{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

For a moving coil galvanometer of ordinary type

$$k\theta = \mathrm{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

A being measured in amperes.

If this deflection be obtained by connecting a resistance in series with the galvanometer and the battery used to charge the condenser, we have

$$\frac{\mathrm{A}}{\mathrm{E}} = \frac{1}{\mathrm{R}}$$

where R is the total resistance including that of the galvanometer, so that the formula for the capacity becomes

$$\mathrm{C} = \frac{1}{\mathrm{R} \tan \theta} \frac{\mathrm{T}}{\pi} \sin \frac{\alpha}{2}\left(1 + \frac{\lambda}{2}\right) \quad . \quad . \quad . \quad (24)$$

If the angles α and θ be small, the expressions for $\tan \theta$ may be replaced by the deflection on the scale d_1, and $\sin \frac{\alpha}{2}$ may be replaced by half the deflection d_2, so that the final form of the formula becomes

$$\mathrm{C} = \frac{1}{\mathrm{R}} \frac{d_2}{2d_1} \frac{\mathrm{T}}{\pi}\left(1 + \frac{\lambda}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Commutator Methods.

A convenient method for measuring capacities with direct current is to use a rotating commutator.

A practical form of commutator is that devised by Fleming and Clinton, a full description of which will be found in Dr. Fleming's "The Principles of Electric Wave Telegraphy and Telephony," or in his "Wireless Telegraphist's Pocket-Book." *

Another form is that described by Rosa and Grover, in their

* Published by the Wireless Press.

paper on "The Absolute Measurement of Capacity," in the Bulletin of the Bureau of Standards, Reprint No. 10.

The Fleming-Clinton commutator consists of three metal barrels, which are mounted on the same shaft, but insulated one from the other. A metal brush rubs on each barrel.

The two outer barrels, A and B, have each four projecting lugs, which interleave with the centre barrel C, which has eight projections. This barrel C is merely to allow the contact brush, alternately in contact with A and B, to run smoothly.

The diagram of connections is shown in Fig. 37, in which it is seen that the condenser is alternately charged to the potential of the battery V, insulated, and discharged through the galvanometer, the whole process being repeated at the rate of about 100 times per second, according to the speed of the barrel. This gives a constant deflection in an ordinary moving coil or other galvanometer, since the period of the instrument is always much greater than $\frac{1}{100}$th of a second. A sensitive galvanometer may therefore be used.

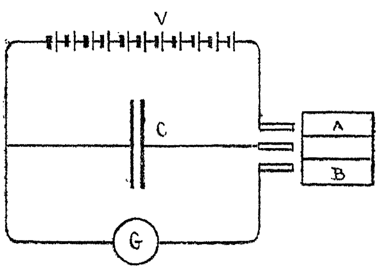


FIG. 37.

The capacity is determined from the formula

$$C = \frac{A}{nV} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

C = capacity in microfarads,
A = current in microamperes corresponding to the deflection of the galvanometer,
n = number of times the condenser is charged per second,
V = potential of charging battery in volts.

n is equal to four times the number of revolutions per second of the commutator, as the barrels A and B have four lugs each.

To determine n, a wheel with 100 teeth engages with a worm on the commutator shaft, so that it makes 1 revolution for 100 of the barrel, and a projection on it causes a bell to ring.

The commutator is best driven by an $\frac{1}{8}$ or $\frac{1}{4}$ H.P. motor, which should be supplied by a steady supply such as that of an accumulator circuit.

The voltage used to charge the condenser should be of from 100 volts downwards according to the capacity to be measured. Resistances to protect this battery against accidental short-circuit are desirable. These must not be of too high a value, or else the condenser will not become fully charged in the time.

The capacity due to the commutator itself must be measured and deducted from the readings for the condenser.

Maxwell's Method for Measuring Capacity.

A condenser with a commutator acts like a resistance, as will be seen from the formula given above.

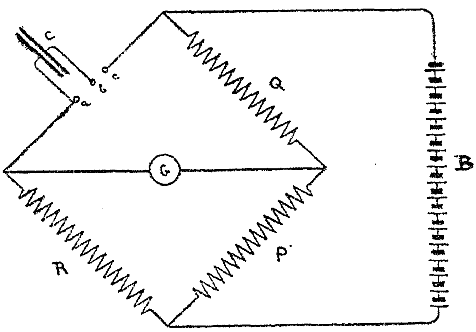


FIG. 38.

By including the combination in one arm of a Wheatstone bridge, the capacity can be found, the method being due to Maxwell.

The diagram of connections is shown in Fig. 38.

The capacity is given by

$$\frac{1}{nC} = \frac{R(P+Q+B)+BP}{P(Q+B)+(G+R)(P+Q+B)}\left\{G+Q+\frac{Q}{P}(G+R)\right\} \quad (27)$$

n being the frequency of the commutator.

When the resistance of the battery is so small that it may be neglected in comparison with the other parts of the circuit, the formula becomes

$$\frac{1}{nC} = \frac{RQ}{P} \times \frac{1+\dfrac{GP}{Q(G+R+P)}}{1-\dfrac{P^2}{(G+R+P)(P+Q)}} \quad . \quad . \quad (28)$$

When the capacity is very small, R + Q will be large compared with P. In this case the formula reduces to

$$\frac{1}{nC} = \frac{RQ}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (29)$$

If the resistances be in ohms, C will be in farads.

This method is the one used for the absolute measurement of capacity. In order that the factor by which $\frac{RQ}{P}$ in the above

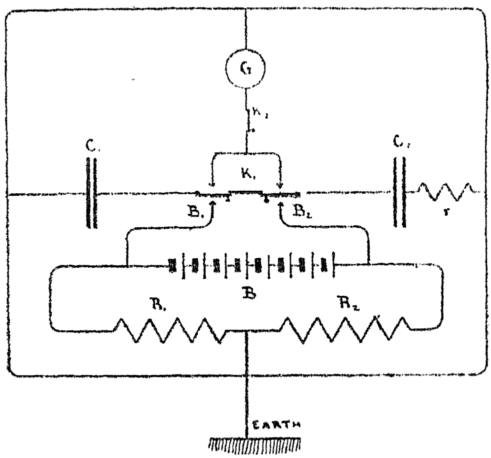


Fig. 39.

formula, No. (28), is multiplied should be as near unity as possible, the resistances of the battery and galvanometer should be small. Also P should be as small as convenient, and Q should be larger than R.

Drs. Rosa and Grover,* in a paper on "The Absolute Measurement of Capacity," show how this factor depends on these various resistances, and give the precautions to be observed for accurate results.

The Method of Mixtures.

In this method the two condensers to be compared are charged by means of a battery, the relative potentials to which they are charged being determined by the ratio of the resistances R_1 and R_2 (see Fig. 39). The condensers are next discharged against each other,

* Bull. Bur. Standards, Reprint No. 10.

and any free charge remaining is then discharged through the galvanometer. The ratio of R_1 and R_2 is varied until there is no discharge through the galvanometer.

This condition is given by

$$R_1C_1 = R_2C_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

For condensers which show absorption, the apparent capacity will depend both on the length of the charging period and of the time of mixing before discharging through the galvanometer. Hence for consistent results these processes must be carried out at definite time-intervals, and the condensers must be completely discharged before another test is made.

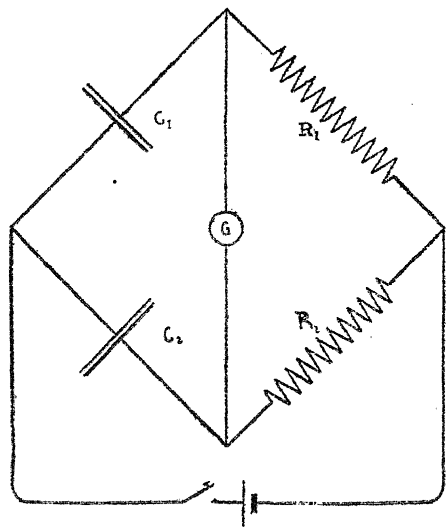


FIG. 40.

This will give accurate results, and the capacity so found is termed the "Acyclic Capacity" for the given conditions, but the process is slow.

Mr. H. Curtis * describes a special apparatus by which these operations can be carried out periodically, and terms the capacity found in this way the Cyclic Capacity.

The capacity found by this method will be slightly larger than the acyclic capacity, since the total residual charge will consist of portions from each of a number of previous charges.

De Sauty's Method.

The connections for this method are given in Fig. 40. The ratio of the resistances is varied until there is no sudden deflection of the galvanometer, when the battery circuit key is closed.

The formula is

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

* Bull. Bur. Standards, vol. 6, No. 4. Reprint No. 137.

Metre Wire Bridge.

When exact measurement of capacity or power factor is required using alternating currents, then the supply voltage must have a pure sine wave form. In Chapter VI. are described wave-form sifters for obtaining a sine wave supply from a generator whose wave form is not a pure sine curve, and also some forms of hummers and other interrupters which generate a sine wave supply.

When only approximately accurate values of capacity are required, however, the alternating-current supply may be replaced

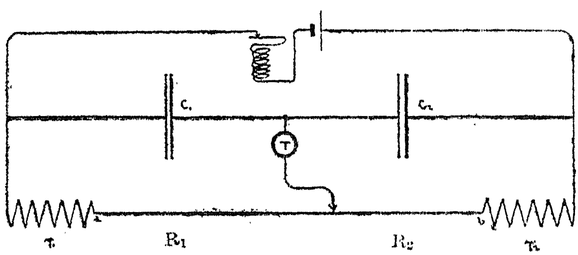


FIG. 41.

by a buzzer giving intermittent direct current. Since the expression for the power factor includes the frequency of the voltage (which is assumed to be of sine form), it is not possible to *measure* the power factor by this means, but the relative losses in condensers may be *compared* by using the Wien bridge and inserting a resistance in series with the condenser.

For rough purposes a convenient capacity bridge is formed by making the resistances R_3 and R_4 in the form of a metre wire, and using one or more mica condensers of fixed capacity as standards. The diagram of connections is as shown in Fig. 41, and the scale of the metre wire may be calibrated to read capacities direct. When used to measure capacities of condensers with power factors much larger than those of the standard, it will be necessary to insert a resistance in series with the latter in order to obtain an exact balance, as indicated by silence in the telephone, but for condensers of the same quality as the standard this will not be necessary.

If a simple metre wire be used for R_3 and R_4, the scale of capacities at the high end will be crowded, and therefore of little use for measurement. If a resistance of one-eighth or one-tenth of the value of

the metre wire be inserted in series with it, the whole of the scale becomes available for measurement, and the graduations are more open.

A similar resistance may be inserted at the other end, as the bridge is not reliable for capacities whose values are small fractions of that of the standard.

CHAPTER V

HIGH-FREQUENCY MEASUREMENTS

In making measurements with alternating currents, the methods used are divided into two main groups: (*a*) those intended for frequencies up to say 3000 cycles per second, which include the range of frequencies used for the transmission of power, and also the frequencies used in making investigations of apparatus for telephone and cable telegraphic work; (*b*) and those methods suitable for the frequencies used in wireless telegraphy, which range from ten million to ten thousand cycles per second.

Investigations have been carried out with frequencies beyond these ranges, but they are for the most part not concerned with the objects of this book—the determination of inductance and capacity.

In the previous chapters the methods of measurement suitable for the lower group of frequencies have been given; in this chapter special methods for high frequencies are to be found.

Inductance.

The high-frequency inductance of a coil may be measured in the following manner.

Connect the coil in parallel with an air condenser or some other condenser of which the high-frequency capacity is known.

Set up electric oscillations in the circuit, a convenient method being to connect a high-note shunted buzzer and cell across the coil. Measure the wave-length of the oscillations by some form of wavemeter.

If the capacity of the condenser be so large that in comparison the self-capacity of the coil may be neglected, the inductance will be given by the formula

$$\lambda_m = 1885\sqrt{LC} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where λ_m is the wave-length of the oscillations in metres,

L is the inductance in microhenrys,

C is the capacity in microfarads.

Self-Capacity of a Coil.

The self-capacity of the coil can be measured by a method described by Prof. G. W. O. Howe.*

The natural frequency of the coil can be determined by bringing it near a circuit in which high-frequency currents are maintained, and adjusting the frequency till the coil resonates to it. Resonance can be detected by noting when a helium or neon or other suitable vacuum tube glows brightest when held near the coil.

The capacity of the coil is then increased. If a calibrated variable condenser of sufficiently small value be available, this may be connected in parallel with the coil.

Prof. Howe increased the capacity by means of brass spheres, placed at various distances apart, connected to the ends of the coils.

The squares of the wave-lengths are plotted against the capacities of the settings of the added condenser, and will be found to lie on a straight line. This line is produced to cut the axis along which the capacities are plotted. The intercept gives the self-capacity of the coil with an accuracy sufficient for all practical purposes.

Capacity.

The high-frequency capacity of a condenser can be determined in a manner similar to that given for an inductance if a standard inductance be available. For a small condenser a convenient method is to substitute a variable air condenser for the one under test, and adjust till the same wave-length is obtained. The result is then independent of the self-capacity of the coil used, but a correction is necessary if the inductances of the condensers be different.

Coupling between Inductive Circuits.

If a circuit having inductance and capacity has an alternating current sent through it by any means, then, on bringing a second circuit with inductance and capacity near the first, an alternating E.M.F. will be induced in it.

* "The Calibration of Wavemeters for Radio-Telegraphy." *Proc. Phys. Soc.*, vol. 24, p. 251 (1912).

If the constants of the circuits are adjusted so that their natural frequencies are the same, and the alternating current in the first circuit, before the second is brought near, be of the same frequency, then it will be found that the currents in the coupled circuit may have two frequencies, both different from the original one.

For high-frequency circuits as used in wireless telegraphy, it is important to know what these are.

The relationship between the frequencies is given by

$$\frac{n_1^2 - n_2^2}{n_1^2 + n_2^2} = k = \frac{M}{\sqrt{L_1 L_2}} \quad . \quad . \quad . \quad . \quad (2)$$

where n_1, n_2 are the two frequencies existing in the coupled circuits, and k is termed the coefficient of (electromagnetic) coupling between the circuits.

From the equation it is seen that if the self and mutual inductances are known or measured, the frequencies can be calculated, or *vice versâ*.

If the natural frequency of the circuits be high as in wireless telegraphic circuits (say from 10,000 to 10,000,000 cycles per second), then by measuring the two frequencies on a calibrated wave-meter the coupling is obtained. If the self-inductances be known, the mutual inductance can be calculated. This method is very convenient for such circuits.

Circuits with Distributed Inductance and Capacity.

The wave-length of a circuit in which the inductance and capacity are localised is given by the formula

$$\lambda_m = 1885\sqrt{LC}$$

the units being metres, microhenrys, and microfarads respectively.

For a circuit in which either part or the whole of the inductance and capacity is distributed over the same unit, as they are in an aerial, a different formula must be used, since the effective values differ from the measured or calculated values of these quantities.

When the aerial is tuned to some other than its natural wave-length, by adding inductance or capacity (or both), we have a circuit consisting partly of distributed and partly of localised inductance and capacity, and the wave-length must be calculated from a formula which takes account of the ratio of the localised to distributed values.

Dr. L. Cohen * has given a formula by which the wave-length of an aerial with an inductance in series may be determined for the case of an aerial with *uniformly* distributed inductance and capacity.

This formula may be put into the form

$$\lambda = \frac{1885\sqrt{L_0C_0}}{Q} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$\cot Q = Q\frac{L_1}{L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$Q = \frac{\pi}{2}\frac{\lambda_0}{\lambda_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where L_1 is the added inductance in microhenrys,
L_0 is the inductance of the aerial,
λ_0 is the natural wave-length of the aerial in metres,
λ_1 is the wave-length of the aerial when the inductance L_1 is in series.

By plotting graphs of $y_1 = \cot Q$ and of $y_2 = \frac{L_1}{L_0}Q$, the points of intersection give the values of Q to insert in the formula to obtain the wave-length of the aerial. It will be found that there will be several points of intersection for any given value of $\frac{L_1}{L_0}$, since the curve for cot Q is that of a periodic function. The first intersection is the fundamental wave-length, and in what follows only this fundamental is considered.

This formula is evidently suitable when the ratio of the aerial inductance to added inductance is known. Examples of its use will be found in Dr. Fleming's "Wireless Telegraphist's Pocket-Book." †

A useful application of the formula would be to obtain the inductance of an aerial by measurement of its wave-length when loaded and not.

The author ‡ has worked out a table, given on page 132, by which the ratio of these inductances can be determined if the two wave-lengths of the aerial, loaded and unloaded, are measured, the inductance added being known.

* *Electrical World*, Jan. 30, 1915.

† Published by the Wireless Press, Ltd.

‡ "The Calculation of Wave-lengths of Aerials," *The Wireless World*, March, 1916.

Dr. W. H. Eccles, in an article in "The Year-Book of Wireless Telegraphy and Telephony, 1916," has given a number of "abacs," or diagrams, by which the same ratio can be read off.

The method of measurement consists in obtaining the natural wave-length of the aerial, which may be done by connecting a small ignition coil across a small spark gap to which the aerial and earth wires are connected.

The wave-length can be read off on a calibrated wavemeter.

Next an inductance of known value is connected in series with the aerial, and the wave-length again measured. In this case a more convenient method of exciting the aerial is to connect a high-note buzzer across the inductance. This enables a small coil of closely wound turns to be used, which would be subjected to risk of puncture between turns if the high voltage of an ignition coil were used.

The ratio of the wave-lengths is then found in the table, and the corresponding ratio of the inductances multiplied by the known value for the coil in use gives the inductance of the aerial.

In the table L_0 is the inductance, and λ_0 the natural wave-length of the aerial itself; L_1 is the added inductance, and λ_1 the resulting wave-length; λ_c is the wave-length calculated by the formula $\lambda = 1885\sqrt{LC}$, and λ_m is the actual measured wave-length.

The second column gives the ratio of the wave-length of the loaded aerial to the natural wave-length of the unloaded aerial, and the third column gives the reciprocal ratio.

In the fourth column the values of Q are given, and in the fifth column the ratio $\frac{\lambda_c}{\lambda_m}$ of the wave-length calculated from the ordinary formula to the true wave-length is given. It will be noticed that as the value of $\frac{L_1}{L_0}$ increases this ratio becomes more and more nearly equal to unity, so that for large values the formula $\lambda_m = 1885\sqrt{LC}$ will give approximately correct results without allowing for the distributed nature of the inductance.

A similar table may be made for the case of an aerial tuned to another wave-length by a series condenser.

In this case the formula is *

$$\lambda_m = \frac{1885\sqrt{L_0C_0}}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

* Eccles, "A Handbook of Wireless Telegraphy and Telephony."

where $$\tan S = -S\frac{C_1}{C_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$S = \frac{\pi}{2}\frac{\lambda_0}{\lambda_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the table No. 14, constructed therefrom, can be used in a similar manner to that for inductance.

It will be noticed that the tables give only the values for the fundamental wave-length to which the aerial will be tuned by the given inductance or capacity, since, for purposes of measurement, these are alone required.

The formulas and tables are strictly applicable only to aerials of which the inductance and capacity per unit length is constant for the whole length of the aerial; but on working the capacity of a number of aerials which do not exactly comply with this condition for which figures were available, the author has found very good agreement between the results given by this and other methods. The aerials included twin and cage wire L and T aerials and one umbrella aerial.

In the *Electrical World* for Jan. 15, 1916, an article by Mr. A. F. Buchstein deals with the case of two aerials in series, but the equations obtained are very complicated and require several measurements for each test.

EXAMPLES.—An aerial of natural wave-length 300 metres gives 924 metres when an inductance of 875 microhenrys is connected in series. Required its inductance and capacity.

Ratio of wave-lengths $\frac{\lambda_1}{\lambda_0}$, $\frac{924}{300} = 3{\cdot}08$.

From table 13 the ratio of $\frac{L_1}{L_0}$, which corresponds to this is 3·5.

Hence inductance of aerial is

$$\frac{875}{3{\cdot}5} = 250 \text{ microhenrys}$$

The natural wave-length of the aerial is given by

$$\lambda = \frac{1885\sqrt{LC}}{1{\cdot}57}$$

since 1·57 is the value of Q which corresponds to $\frac{L_1}{L_0} = 0$

Hence $$(300)^2 = \frac{(1885)^2}{(1{\cdot}57)^2} \times 250 \times C$$

whence $$C = {\cdot}00025 \text{ mfd.}$$

To what wave-length will the above aerial be tuned if a capacity of $\cdot0001$ mfd. be connected in series ?

We have $$\frac{C_1}{C_0} = \frac{\cdot0001}{\cdot00025} = \cdot4$$

for which $$\frac{\lambda_1}{\lambda_0} = \cdot661$$

whence $$\lambda_1 = 300 \times \cdot661 = 198 \text{ metres}$$

CHAPTER VI

APPLIANCES FOR USE IN MEASURING INDUCTANCE AND CAPACITY

Resistance Coils for Alternating Currents

In making measurements of inductance and capacity, especially by methods involving the use of alternating currents, it is necessary that any resistance coils used in the test should not influence the result by reason of their inductance or capacity.

The subject of suitable resistance coils for alternating-current work has been investigated by Messrs. Curtis and Grover, and the results of their work, including the method of measuring the inductance of such coils, is given in two papers in the Bulletin of the Bureau of Standards.*

An ideal coil is one which does not change in resistance with the frequency, and of which the phase angle is zero.

The change in resistance with frequency is due (1) to the skin effect, and (2) to the inductance and capacity of the coil.

The phase angle depends on the inductance and capacity.

Messrs. Grover and Curtis show that up to 3000 cycles the skin effect will have no effect on the resistance of a coil, since even for a straight wire the diameter must be 2 mm. for a change of 1 part in 100,000, and the effect of a coil whose inductance is less than that of the equivalent straight wire (as it is for all resistance coils) will be proportionately less.

The very high frequencies used in radio-telegraphy are not considered in this connection, since the methods suitable for measurement are different, and the order of accuracy at present attainable is not so great.

There will be some change of effective resistance with frequency due to the inductance and capacity of the winding, since the impedance due to these depends on the frequency.

* Bull. Bur. Standards, vol. 8, No. 3 (1912). Reprints Nos. 175 and 177.

The phase angle of the coil is given by

$$\tan \theta = \frac{p(L - \frac{1}{3}CR^2)}{R}$$

when L and C are both small.

If $L - \frac{1}{3}CR^2 = 0$, the phase angle will be zero, and the change of effective resistance with frequency will be negligible, if C itself is small.

For low-value coils such as 10 ohms and less, the inductance is of greater importance than the capacity, but for high-value coils the reverse is the case.

To reduce the capacity of large-value coils it is better to wind several smaller-value coils and connect in series.

High-value coils are now made by winding the wire inductively on thin mica cards, by which the capacity between turns is reduced from what it would be for a bifilar cylindrical winding, but there will be a greater capacity between different cards in the same box. This method was first described by Prof. Rowland.

Another method, due to Messrs. Duddell and Mather, is to wind the wire as the woof in a silk warp.

In using resistance boxes for alternating-current work, the circuits should be arranged, wherever possible, so that there is not a large number of idle coils in the box. The capacity of such coils to earth and to the rest of the apparatus may influence the results.

The great advance made in recent years in alternating-current measurements has had the effect of putting on the market resistance units specially designed for such tests.

In some tests, such as the measurement of small inductance, it is preferable to have the resistance units arranged so that their total inductance is constant for various values of resistance. For large resistances this is accomplished by arranging that a copper winding of equal inductance is put in circuit for every resistance cut out. This can be done by means of a special switch.

For small resistances Mr. Campbell has designed a constant-inductance rheostat consisting of a copper wire and a manganin wire laid parallel round the rim of a slate disc, a sliding contact being arranged so that as much of the copper wire is put in circuit as there is manganin wire cut out.

Another form of low-resistance rheostat is due to Messrs. Wenner and Dellinger. It consists of a U-tube of which the limbs are of unequal diameter. The tube contains mercury, and copper rods

VARIABLE INDUCTANCE, D COIL TYPE.

(*Page* 103.)

are arranged so that they may be pushed down to short-circuit more or less of the mercury column. The larger limb is for a fine adjustment, and the positions of the copper rods can be read off on a scale.

Low resistance standards, to carry heavy currents, are required for use with the A. C. Potentiometer and other instruments. There are several patterns now made which have been especially designed for alternating-current work.

Variable Inductances and Mutual Inductances.

The Campbell Mutual Inductometer has already been described.*

A useful variable self or mutual inductance is the D coil type. It consists of two sets of coils, each set consisting of two flat coils wound in a semicircular or D shape, and fixed back to back on a disc of wood, ebonite, etc. The two sets of coils are placed face to face, and can be rotated relatively to each other round the axis of the discs, by which the mutual inductance between them varies, or, if the coils be joined in series, the self-inductance of the combination varies. The scale is uniform over a wide range.

Standards of Inductance.

Where standards of the greatest accuracy are required, they are usually wound in grooves cut in the rim of a marble disc. Marble is the best material of which to construct the former, but any material used must be examined to see that it contains no iron or other material by which the inductance might be influenced.

For ordinary purposes thoroughly seasoned wood such as mahogany may be used. Ebonite is largely used as formers for coils, but has the disadvantage of altering in dimensions with temperature, which renders it unsuitable for standard coils of the highest accuracy.

In order that the inductance of a coil should not vary appreciably with the frequency, it should be wound of stranded fine insulated wire.

Standards of Capacity.

For small values of capacity, well-made *air condensers* form the best form of standard.

* See page 72.

The coefficient of change of capacity with temperature and pressure is small and can easily be measured, and the phase angle is for most purposes sensibly equal to zero.

For capacities above about ·02 mfd. the air condenser becomes too bulky for convenient use, and the next best form of standard is the mica condenser.

Mr. Curtis * has investigated the suitability of *mica condensers* as standards of capacity, and his results are summarised as follows :—

The temperature coefficient of a mica condenser may be made very small by subjecting it to pressure whilst warm.

The effect of changes in atmospheric pressure is small, though in some cases they are not negligible.

Condensers kept in a vacuum at constant temperature can be relied on to a few parts in a hundred thousand. Condensers exposed to the variation of atmospheric temperature and pressure show unaccountable variations of capacity of two or three parts in ten thousand.

The capacity measured varies with the method of test. For alternating currents it varies with the frequency, being smaller for the higher frequency. For direct current the capacity varies considerably with the time of discharge, being greater with the longer time. The measurement of the capacity by direct current cannot be used for predicting the capacity for alternating current, but, in general, a condenser which shows only small variation for different conditions using D.C. will show small variations for A.C. of different frequencies, and *vice versâ*.

The capacity of a condenser with A.C. of infinite frequency is the same as that with D.C. when the time of discharge is infinitely short, and this capacity is termed the **Geometric capacity**, since it is a function of the dimensions and dielectric constant.

The geometrical capacity is obtained by plotting the apparent capacities as ordinates and the period (reciprocal of the frequency), as abscissæ, and extending the curve to the point of zero period. For actual measurements at high frequencies the inductance of the leads has to be allowed for, and at very high frequencies the fact that the charge may not be evenly distributed over the metal plates will have to be taken into consideration. Hence the measurements are usually carried out between 100 and 1200 cycles, for which these effects are negligible.

* Bull. Bur. Standards, vol. 6, No. 4. Reprint No. 137.

For direct current the apparent capacity is plotted against the time of discharge, which must be short and definitely known.

Condensers made with silvered mica show a variation of capacity with the applied voltage, which is considered to be due to flakiness of the silver which makes imperfect contact with the mica at places.

The Duddell 2000-Cycle Alternator.*

This machine is very useful as a source of supply for the various bridge methods of measurement described, and can also be used to supply the current when making tests with the alternating-current potentiometer.

The output rises from a minimum of 10 watts at the lower part of the working range to about half a kilowatt at the higher frequencies.

The rotor has 30 salient poles which carry the field windings, and runs at 8000 revs. per min. for 2000 cycles.

In order to obtain a smooth wave form the stator is not slot-wound, but consists of a smooth ring provided with a gramme ring winding. The wave form is practically a sine wave, but there is a small third harmonic present which has no influence on the accuracy for most tests.

The Campbell Microphone Hummer.

In this instrument a carbon grain microphone is carried in a fitting at one end of an iron bar, which rests on two supports placed at a quarter of its length from each end.

A polarised magnet is placed under the centre of the bar. The current through the microphone passes through one winding of a transformer, the magnet is connected to a second winding, and the measuring circuit to a third. Condensers are connected in the microphone and magnet circuits, and to prevent electrostatic disturbance an earthing shield is placed between the second and third windings on the transformer.

When supplied with from 6 to 10 volts the hummer gives an alternating-current supply which is practically free from harmonics, and the frequency of which depends only on the physical constants of the iron bar.

There are a number of other forms of interrupters and buzzers, of which particulars must be sought in the makers' catalogues.

* "A 2000 Frequency Alternator," by W. Duddell. *Proc. Phys. Soc.*, xxiv. p. 172 (1912).

The diagram of connections is shown in Fig. 42.

Where alternating current is not a necessity, a simple buzzer may often be used and forms a convenient and compact source of supply.

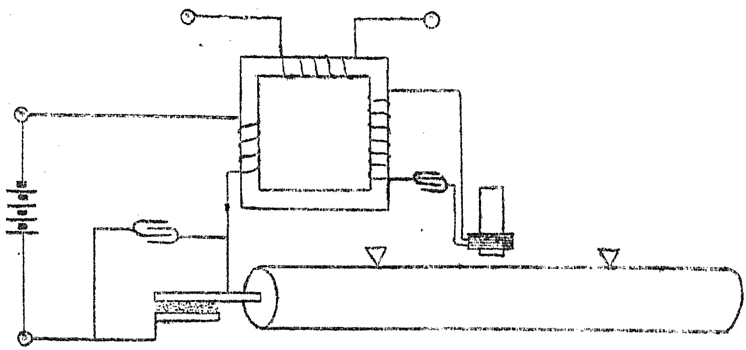


FIG. 42.

A small air core transformer should be used in connecting to the bridge if the latter is of more than one or two ohms resistance, and a shunt should be connected in parallel with the coil to prevent sparking.

Wave-form Filters and Sifters.

In making measurements, with alternating currents, of capacities and inductances, it is, in most cases, essential for the wave form of the supply to be a pure sine curve.

In some cases the equations for the balance of the particular test in use do not contain a term representing the frequency, but even in these cases a sine-wave supply is often essential for accurate work, since the value of the inductance or capacity is not usually constant for all frequencies, whilst there are usually subsidiary factors, to be reckoned with in the test, which are directly dependent on the frequency.

Alternating wave forms generated by ordinary means which depart from the sine curve can be resolved into a number of components, each of a sinusoidal form, but of frequencies which are in the ratio of 3, 5, 7, etc., of the principal or fundamental wave.

Alternating-current generators, either in the form of machines or certain types of hummer, etc., can be obtained, which give a supply of practically pure sine wave form, some of which are described in this chapter, but in many cases the absence of these special forms of apparatus makes it necessary to adapt other types to the purpose.

This is done by wave filters or wave sifters which are resonating circuits containing inductance and capacity, by which the desired frequency can be selected from the generator and passed on to the testing apparatus or, on the other hand, by which a certain frequency may be suppressed between the two circuits.

In Fig. 43 is shown the apparatus used by Fleming and Dyke in their investigation on the Power Factor and Conductivity of Dielectrics.*

The alternator used had a fundamental frequency of 920 periods, the wave form not being sinusoidal. By means of the filter a pure

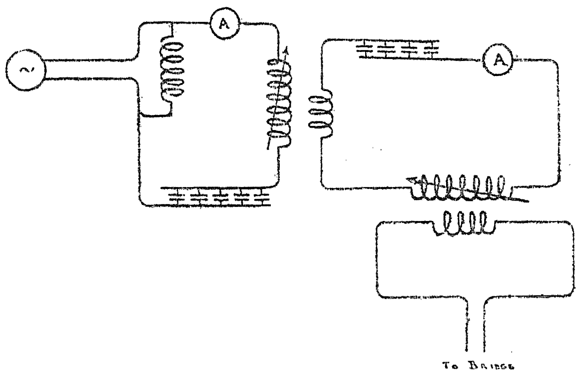


FIG. 43.

sine wave of either 920, 2760, or 4600 periods could be supplied to the apparatus.

A number of variable inductances were made, each consisting of a pair of coils joined in series, one of which could be slipped over the other so that the inductance was continuously variable between certain limits. The actual ranges of the coils constructed were: 1380 to 2370; 683 to 1163; 150 to 536 microhenrys for the first filter, and 1400 to 2400 microhenrys for the second filter.

The condensers associated with these coils were of paper, and were adjusted to 20, 4, or 2 microfarads for the three frequencies for the first, and from 8·25 to ·25 microfarads for the second circuit.

The circuits were coupled to each other and to the bridge used for the measurements by means of air-core transformers consisting of coils which slipped inside the variable inductances. It was found

* *J.I.E.E.*, xlix. (1912), p. 323.

necessary to connect a coil across the alternator terminals to provide a complete circuit which the selected component of the wave could flow.

The double filtration was found to be essential in order to obtain a pure sine wave.

To tune a set of circuits of this description, the capacity and inductance are first set to the values given by a rough preliminary calculation, and finally adjusting till the alternating-current ammeters in the various circuits give their maximum readings.

Mr. A. Campbell * has described a circuit by which one component of a wave form may be suppressed. The arrangement is shown in Fig. 44. If the mutual inductance be M henrys and the

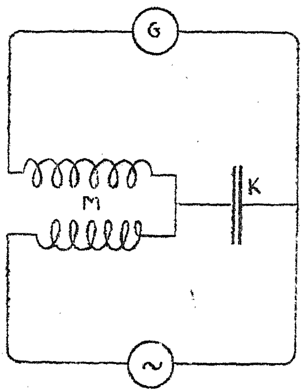


FIG. 44.

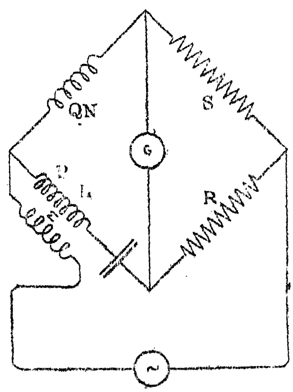


FIG. 45.

capacity K microfarads, then for $4\pi^2n^2MK = 1$ no component of frequency n will pass through the circuit at G.

If the condenser has any absorption, the above relationship will not apply unless compensated for by another condenser placed across the secondary of the mutual inductometer.

Mr. Campbell gives the following table of approximate values of K and M suitable for sifting out various frequencies :—

n cycles per second.	M. millihenrys.	K. microfarads.
10	6400	40
50	1000	10
100	128	20
500	10	10
1000	25·6	1
5000	1·0	1
10000	·256	1

* *Proc. Phys. Soc.*, xxiv. p. 107 (1912), "On Wave-form Sifters for Alternating Currents."

From this table it will be seen that for low frequencies both M and K are large. To obviate the use of such apparatus the arrangement shown in Fig. 45 is given by Mr. Campbell for these cases. R and S are non-inductive resistances, P and Q are the resistances of the coils, whose self-inductances are L and N respectively.

Then if $PS = QR$,

$$LS = NR$$

and

$$4\pi^2 n^2 MK = \frac{S}{S + R}$$

no current of frequency n will flow through G.

The Telephone.

A telephone receiver is one of the most convenient detectors for many of the bridge tests using alternating or interrupted current, if the frequency is between 100 and say 3000 cycles.

There are many patterns now on the market, some being of a high order of sensitivity, and they are obtained wound to all resistances from 60 to 8000 ohms per head.

Where it is desired to use a low-resistance telephone with a bridge of high resistance or impedance, a suitable transformer may be used. These transformers usually have an iron core, and the two circuits may have condensers connected to tune them to the frequency of supply.

When such a transformer or sensitive telephones are in use, care must be taken that they do not pick up extraneous noises by induction from the supply mains or other circuits.

When the supply is not a pure sine wave, the balance point may be difficult to judge, due to the higher harmonics giving a sound in the telephone.

Frequency Indicators.

Where the method of test involves the frequency, this must be neasured in some suitable manner.

When the supply is from an ordinary alternating-current machine, one of the usual pattern frequency meters may be used, but since the equations usually contain a term representing the square of the frequency, it is necessary that the meter should be finely graduated.

The vibrating reed type has been made for a frequency of 1500 cycles, but this is as high as it is possible to use it. Above 1000 cycles a bridge method of measurement is preferable.

Mr. A. Campbell has shown that the circuits shown in Figs. 44 and 45 can be used as frequency meters, since there is no current through the circuit at G when the relationship $4p^2n^2MK = 1$ holds. The variable mutual inductance can be calibrated to read n directly.

There are a number of other bridge methods which can be used for the measurement of frequency.

The Vibration Galvanometer.

The vibration galvanometer is an instrument arranged so that the suspension is capable of vibrating, the natural period of vibration being adjustable so that it may be made to coincide with that of the current passing through it.

When so adjusted the deflection for a given current is very much greater than when slightly out of tune. In many cases it may be used in circuits in which the supply is not a pure sine wave, as it will indicate the balance for the frequency to which it is tuned without being affected by the other components of the wave.

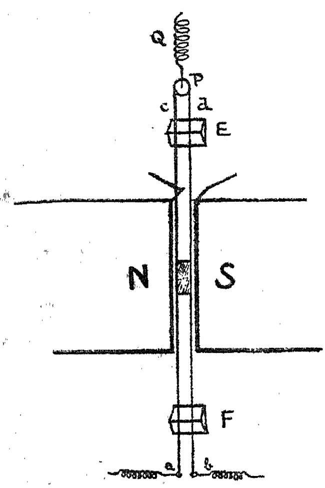


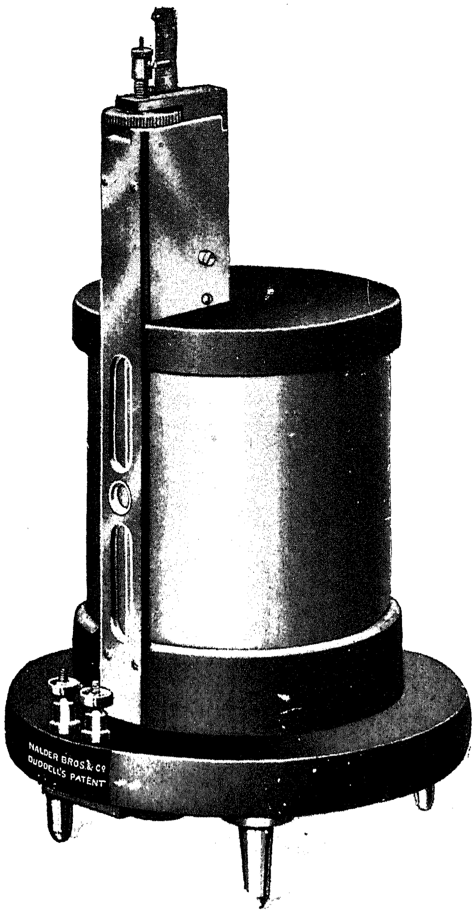
FIG. 46.

The Duddell Vibration Galvanometer.*

In this instrument a phosphor-bronze wire *abcd* (Fig. 46) passes over a small pulley P, and is kept tight by means of a spring Q, the tension of which may be varied by means of a milled head. The wires pass over two bridge pieces E, F, which can be moved closer to or farther from each other by means of a right-and-left-handed screw. This varies the length which is free to vibrate. The wires lie in a strong magnetic field produced by a permanent magnet NS, and a small mirror is fixed to the two wires at the centre of the vibrating length.

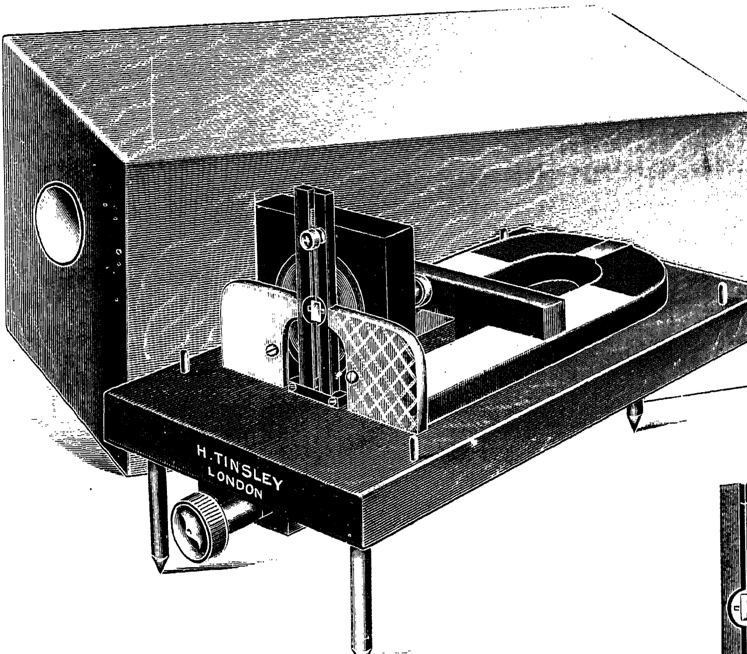
By varying the length of wires between the bridges E, F, or its

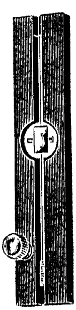
* "On a Bifilar Vibration Galvanometer," W. Duddell. *Proc. Phys. Soc.*, Feb. 1910, xxi. 774.



DUDDELL VIBRATION GALVANOMETER.

(*Page* 110.)





TINSLEY VIBRATION GALVANOMETER.

(*Page* 111.)

tension (or both), their natural period of vibration may be varied so that it may be brought into resonance with the alternating current supplied. The standard instrument may be tuned to frequencies between 100 and 1800 cycles per second.

In his paper Mr. Duddell gives curves for the sensitivity of the galvanometer at various frequencies, and states that, although it is possible to tune the instrument to a given frequency, using various combinations of length and tension, the sensitivity per microampere of current passing through it is very nearly the same as long as the wires are longer than the pole pieces.

The resonance is very sharp, that is, the deflection for a given current is greatly reduced when the galvanometer is only a little out of tune, so that it is necessary to carefully tune it to the frequency of the supply when making a test.

The sensitivity in millimetres deflection per microampere for a scale distance of one metre, is approximately

Cycles.	
100	57 mm. per microampere
200	27 ,, ,,
400	16 ,, ,,
600	9 ,, ,,

which are average figures taken from the curves in the paper.

It will be seen that the sensitivity varies inversely as the frequency.

Owing to the fact that the wires are vibrating in a magnetic field, a strong back E.M.F. is produced, and this reduces the sensitivity as a voltmeter.

The sensitivity as a voltmeter cannot therefore be calculated from the current sensitivity and the ohmic resistance. There is some advantage, in measuring small potential differences, to use as short wires as possible.

A paper by Dr. Haworth on the maximum sensitivity of the Duddell Vibration Galvanometer, is published in the *Proceedings of the Physical Society of London*, vol. xxiv. p. 230 (1912).

The Tinsley Vibration Galvanometer.*

This galvanometer is designed to work with alternating-current supplies of from 50 to 100 cycles.

* "A Magnetic Shunt Vibration Galvanometer," by H. Tinsley. *Electrician*, Sept. 13, 1912, p. 939.

It consists of a horizontal, flat, horse-shoe form of permanent magnet, with shaped vertical pole pieces between which the metal frame carrying the suspended magnet and mirror is fixed.

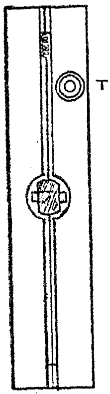


FIG. 47.

The magnet is of soft iron, suspended by a silk fibre kept taut by a small spring (see Fig. 47).

The coil, through which the alternating current to be detected flows, is fixed behind and close to the magnet in a similar manner to the Kelvin galvanometer.

A square iron bar is fitted which can be slid over the limbs of the permanent magnet, thus shunting some of the magnetic field which otherwise would pass between the pole pieces.

By varying the field in this manner the free period of the magnet in the field can be varied, and thus brought into resonance with the supply. The coils can be changed in a moment, so that either a high or low resistance winding may be used, according to the purpose for which it is required.

The following table is given by Mr. Tinsley for the sensitivity at 50 cycles for a scale distance of 1 metre :—

R. ohms.	L. millihenrys.	Impedance at 50 cycles. ohms.	Deflection per Microvolt. millimetres.	Deflection per Microampere. millimetres.
·005	·0034	·0051	16	·08
1·2	·98	1·26	·8	1·0
12	6·9	12·2	·25	3
40	17	41·0	·15	6
250	270	28·5	·06	18
1600	720	1650	·025	40
5800	2800	6000	·012	70
17000	5300	17500	·005	90

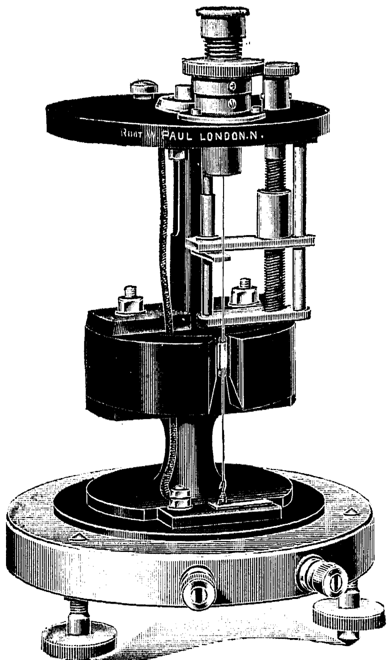
The deflection is directly proportional to the current.

The Campbell Vibration Galvanometer.*

In this instrument a narrow coil is suspended in the field of a permanent magnet, being held by a double bifilar suspension.

The effective length of the upper suspension can be varied by means of a movable bridge piece, and the tension can be varied by a spring which is carried by an adjusting screw.

* *Proc. Phys. Soc.*, xx. p. 630, Dec. 1907; xxv. p. 203, April, 1913.



CAMPBELL VIBRATION GALVANOMETER.

(*Page* 112.)

The instrument can be tuned from about 40 to 1000 periods per second.

In another pattern the coil is suspended by a unifilar strip. This pattern can be tuned from about 10 to 400 periods.

The sensitivity of these instruments, as given in the maker's * catalogue is

BIFILAR GALVANOMETER.

Frequency	50	100	350	750	1000	periods per second
Sensitivity	60	30	3	0·5	0·2	mm. per microampere
Effective Resistance .	500	350	160	52	35	ohms

UNIFILAR INSTRUMENT.

Frequency	30	40	50	100	200	periods per second
Sensitivity	20	20	25	40	10	mm. per microampere
Effective Resistance .	50	40	50	120	70	ohms

The Vibration Electrometer.

Mr. H. L. Curtis has described a vibration electrometer in the Bulletin of the Bureau of Standards, vol. xi. p. 535.

The instrument consists of a small rectangular aluminium vane, which is suspended by a bifilar suspension somewhat similar to that of the Duddell Vibration Galvanometer described on page 110.

The bifilar suspension wires pass over bridge pieces by which their effective length can be varied, and are kept stretched by means of a spring by which the tension can be varied. A constant potential is applied to the vanes by a set of cells.

Four metal plates, P, P, Q, Q, are arranged similarly to the plates of a quadrant electrometer round the vane. The alternating voltage to be detected is connected to these plates.

The amplitude of vibration depends on the damping, which for this instrument is principally due to air damping, so that the whole instrument is used under a bell jar, by which means the air pressure can be reduced to a suitable value.

In his description Mr. Curtis states that it will give a deflection of 1 cm. at 1 metre distance for 10^{-9} amperes.

For any particular adjustment of the instrument the frequency at which resonance occurs depends on the voltage applied to the vane. As this voltage is increased, the frequency is decreased. Increase of the voltage of the vane will increase the sensitivity, the deflection increasing at a greater rate than the voltage, but owing

* R. W. Paul, Catalogue, Section H, p. 690.

to the lowering of the resonance frequency it is not possible to indefinitely increase the sensitivity in this way.

The deflection is inversely proportional to the damping. As the latter is decreased, the range of frequencies over which the instrument can be used is reduced, so that it is not possible to reduce the damping below a certain limit.

The time required for the amplitude of vibration to become constant after the circuit is closed is increased as the damping decreases. Practically the whole of the damping is due to the air, the suspension having very little effect when properly constructed.

The instrument cannot be used on frequencies much above 100 cycles on account of the moment of inertia of the vane.

The connections of the circuit used in testing the instrument are shown in Fig. 48. The original paper gives a number of curves showing the variation in sensitivity, etc., as the constants are altered.

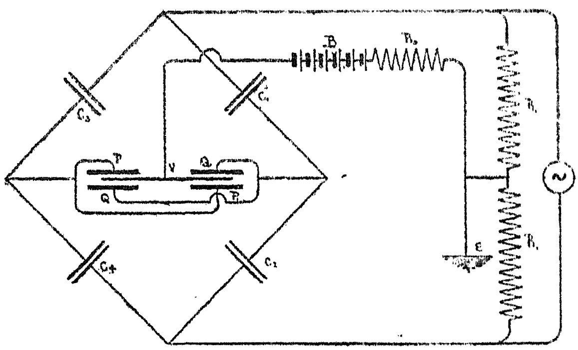


FIG. 48.

The Sumpner Alternating-Current Galvanometer.*

This instrument is essentially a moving coil galvanometer, in which the field is due to an electromagnet excited by an alternating current.

The alternating current applied to the field coils of the electromagnet will produce a core flux N given by

$$V = rA + mN$$

where

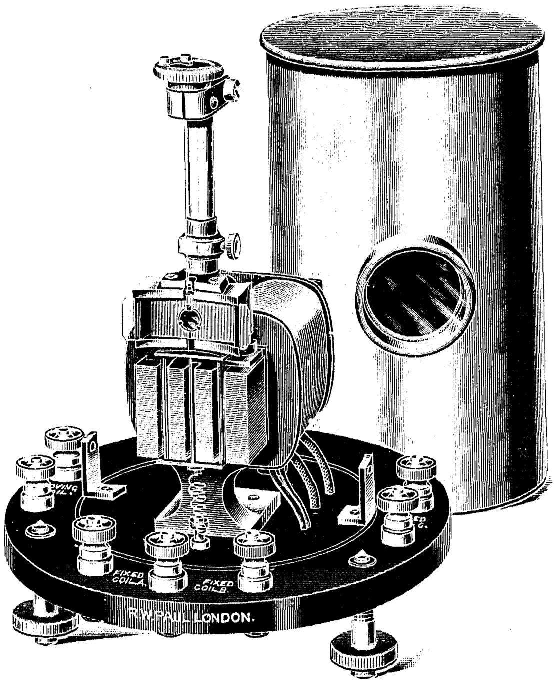
V = applied volts,

r = resistance of coils,

A = current in coils,

m = number of turns.

* "A Galvanometer for Alternating-Current Circuits," by W. E. Sumpner and W. C. S. Phillips. *Proc. Phys. Soc.*, xxii. p. 395 (1910).



SUMPNER ALTERNATING-CURRENT GALVANOMETER.

(Page 114.)

The galvanometer is designed so that for the currents and frequencies used the resistance is negligible in comparison with the impedance, so that the term rA can be neglected. Hence the rate of change of N will be at each instant a measure of V, and this is true whatever the permeability or hysteresis of the core.

The standard pattern of galvanometer is wound with a coil of 2000 turns of fine wire and a coil of 100 turns of thicker wire on each limb of the electromagnet, so that it may be excited by a winding either of 200, 2000, or 4000 turns, the coils being brought out to terminals for this purpose. The magnetisation of the iron will not be too great if the volts be applied at the rate of 20 turns per volt, so that a maximum of 200 volts may be used for excitation.

The galvanometer must, of course, be excited from the same supply as that used for the measuring apparatus.

The moving coil may be connected as the detector in most of the bridge methods of measurement described.

The Ballistic Galvanometer.

There are two forms of ballistic galvanometer in use—the moving magnet, and the moving coil instrument.

The function of a ballistic galvanometer is to measure the quantity of electricity which passes, and for this purpose it is necessary that the time during which the current is flowing should be short compared with the time of swing of the galvanometer. It is also necessary that the damping of the galvanometer should not be large.

Any galvanometer which fulfils these conditions may be used for ballistic work, but some forms have been devised which are specially suitable for this purpose.

In the moving magnet instrument a special form of magnetic needle is used. It consists of a small cylinder with hemispherical top. The cylinder has a saw-cut along its length, and is magnetised with its two poles on the limbs of the horse-shoe type magnet thus formed. The magnet is suspended by a silk fibre between two coils such as are used in the Thomson galvanometer, the controlling field being modified by a bar magnet fixed to the top of the galvanometer case.

To bring the galvanometer to rest after a test has been made, a small coil, preferably without an iron core, is fixed in such a position that when a current is sent through it, it will cause the galvanometer needle to swing.

By means of a tapping key the impulses from this coil are timed so that their effect is opposite to the direction in which the needle is swinging, therefore the amplitude of vibration is reduced, and by careful timing the needle may be quickly brought to rest. It is best to include a variable resistance in the circuit of the coil so that as the amplitude of vibration gets less the impulses from the coil may be made weaker.

The moving coil galvanometer consists of an ordinary D'Arsonval galvanometer without the usual damping arrangement, the coil having a long period of vibration.

The damping of a moving coil galvanometer is effected in either of two ways. In the first the moving coil is placed in a metal tube which is suspended between the poles of the magnet. When the coil swings, the eddy currents in this tube damp the vibrations quickly.

This type cannot be used as a ballistic galvanometer.

In the other type the damping is effected by shunting the moving coil by a non-inductive resistance, the value of which is just sufficient to make the vibration aperiodic. By disconnecting this resistance the galvanometer is converted into a ballistic one. This type of galvanometer can be brought to rest by short-circuiting the coil by a key.

The Ayrton-Perry Secohmmeter.

In making measurements of inductance, using direct current, greatly increased sensitivity is obtained if, instead of a single make or break of the battery circuit sending a single impulse through the galvanometer, a succession of impulses at a rapid rate can be sent.

By including a rotating commutator in the battery circuit, the current can be made and broken at the desired frequency. Since, however, the galvanometer throw is in reverse directions for these two cases, one must be prevented from passing through the instrument by either short-circuiting its terminals or breaking its connection with the bridge.

An Ayrton-Perry secohmmeter is an instrument for performing these operations. It consists of two discs mounted on one shaft, but insulated from it and one another.

A brass segment is let into the rim of each disc, and spring contact brushes arranged to make contact with it.

By one set of brushes the connection between the battery and the bridge is alternately made and broken, and by the other the galvanometer is short-circuited at make and open-circuited at break,

or *vice versâ*, so that a succession of impulses in the same direction are sent through it.

In another form of the secohmmeter the battery and galvanometer connections are both reversed by having two brass segments and four brushes for connecting the battery or galvanometer to the bridge.

When the battery circuit is broken and then made with the connections reversed, the galvanometer impulses are in the same direction. If the galvanometer connections be now reversed, the succeeding impulses will also be in the same direction.

In the secohmmeter arrangements are made so that one set of brushes can be moved through an angle in order that the proper setting can be obtained.

Electrostatic Capacity of the Bridge.

When making tests of capacity by bridge methods, especially if telephone receivers be used as detectors, the result may be affected by the electrostatic capacity of the various parts of the bridge to earth. It is necessary, in order that the capacity of the bridge to earth should not affect results, that the points to which the telephones are connected should be at earth potential, since when this is so no disturbing effect will be transmitted through the capacity between the telephones and observer to earth.

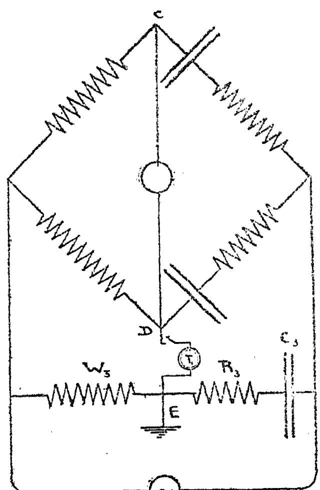


FIG. 49.

In making tests by the Wien bridge the following method was adopted by Mr. Wagner* for bringing the telephone connections to earth potential.

The generator is shunted by a resistance W_3R_3 with a condenser C_3 in series, and the point E earthed so that no current flows in a telephone receiver connected between E and either C or D after the main bridge has been approximately

* "The Measurement of Dielectric Losses by Means of the Wheatstone Bridge," K. W. Wagner, *Electrician*, Dec. 29, 1911, p. 483.

balanced (see Fig. 49). When this adjustment is made, the connection is opened and a final balance is made in the bridge circuit.

In some tests it is necessary to bring some other part of the bridge to earth potential than the detector. Thus, in measuring the inductance of resistance coils, Messrs. Grover and Curtis * used a standard consisting of two parallel straight wires. To calculate the capacity of this arrangement, it is necessary that one wire should be at the same potential above that of the earth as the other is below it.

The adjustment of the shunt resistance and condenser in this case is made so that no current passes through a telephone connected to the mid-point of the wires and earth.

The condenser may be replaced by an inductance at the opposite end of the shunt resistance.

* "The Measurement of the Inductances of Resistance Coils," Grover and Curtis, Bull. Bur. Standards, vol. 8, No. 3. Reprint No. 175.

TABLES

TABLE 1.

NAGAOKA'S FACTORS FOR THE INDUCTANCE OF A SINGLE-LAYER COIL.

$\frac{d}{l}$	K	Log_{10} K	$\frac{d}{l}$	K	Log_{10} K
·00	1·000	·0000	·50	·8181	$\bar{1}$·9129
·01	·9958	$\bar{1}$·9981	·51	·8151	·9113
·02	·9916	·9964	·52	·8120	·9096
·03	·9874	·9944	·53	·8090	·9079
·04	·9832	·9927	·54	·8060	·9063
·05	·9791	·9908	·55	·8031	·9048
·06	·9750	·9890	·56	·8001	·9032
·07	·9709	·9872	·57	·7972	·9016
·08	·9668	·9853	·58	·7943	·9000
·09	·9628	·9836	·59	·7914	·8984
·10	·9588	$\bar{1}$·9818	·60	·7885	$\bar{1}$·8968
·11	·9548	·9799	·61	·7857	·8953
·12	·9509	·9781	·62	·7828	·8936
·13	·9469	·9763	·63	·7800	·8921
·14	·9430	·9745	·64	·7772	·8905
·15	·9391	·9727	·65	·7745	·8890
·16	·9353	·9709	·66	·7717	·8874
·17	·9314	·9691	·67	·7690	·8859
·18	·9276	·9674	·68	·7663	·8844
·19	·9239	$\bar{1}$·9656	·69	·7636	·8828
·20	·9201	·9638	·70	·7609	$\bar{1}$·8813
·21	·9164	·9621	·71	·7582	·8798
·22	·9126	·9603	·72	·7556	·8782
·23	·9090	·9586	·73	·7530	·8768
·24	·9053	·9567	·74	·7504	·8753
·25	·9016	·9549	·75	·7478	·8738
·26	·8980	·9533	·76	·7452	·8723
·27	·8944	·9515	·77	·7426	·8708
·28	·8909	·9498	·78	·7401	·8693
·29	·8873	·9480	·79	·7376	·8679
·30	·8838	$\bar{1}$·9464	·80	·7351	·8664
·31	·8803	·9446	·81	·7326	$\bar{1}$·8649
·32	·8768	·9428	·82	·7301	·8634
·33	·8734	·9412	·83	·7277	·8619
·34	·8699	·9395	·84	·7252	·8604
·35	·8665	·9378	·85	·7228	·8589
·36	·8632	·9361	·86	·7205	·8577
·37	·8598	·9345	·87	·7180	·8561
·38	·8565	·9327	·88	·7157	·8547
·39	·8531	·9310	·89	·7133	·8533
·40	·8499	$\bar{1}$·9294	·90	·7110	$\bar{1}$·8519
·41	·8466	·9277	·91	·7086	·8504
·42	·8433	·9260	·92	·7063	·8490
·43	·8401	·9244	·93	·7040	·8476
·44	·8369	·9227	·94	·7018	·8462
·45	·8337	·9210	·95	·6995	·8448
·46	·8306	·9194	·96	·6972	·8437
·47	·8274	·9177	·97	·6950	·8420
·48	·8243	·9161	·98	·6928	·8406
·49	·8212	·9145	·99	·6906	·8392

TABLE I.—*continued.*

NAGAOKA'S FACTORS FOR THE INDUCTANCE OF A SINGLE-LAYER COIL.

$\frac{d}{l}$	K	Log_{10} K	$\frac{d}{l}$	K	Log_{10} K
1·00	·6884	$\bar{1}$·8379	3·1	·4217	·6250
1·05	·6777	·8311	3·2	·4145	·6175
1·10	·6673	·8243	3·3	·4075	·6101
1·15	·6573	·8178	3·4	·4008	·6030
1·20	·6475	·8112	3·5	·3944	·5959
1·25	·6381	·8049	3·6	·3882	·5890
1·30	·6290	·7986	3·7	·3822	·5823
1·35	·6201	·7925	3·8	·3764	·5757
1·40	·6115	·7864	3·9	·3708	·5691
1·45	·6031	·7804	4·0	·3654	$\bar{1}$·5628
1·50	·5950	$\bar{1}$·7745	4·1	·3602	·5565
1·55	·5872	·7687	4·2	·3551	·5504
1·60	·5795	·7631	4·3	·3502	·5443
1·65	·5721	·7575	4·4	·3455	·5384
1·70	·5649	·7520	4·5	·3409	·5326
1·75	·5579	·7466	4·6	·3364	·5268
1·80	·5511	·7413	4·7	·3321	·5212
1·85	·5444	·7359	4·8	·3279	·5157
1·90	·5379	·7307	4·9	·3238	·5103
1·95	·5316	·7256	5·0	·3198	$\bar{1}$·5049
2·00	·5255	·7206	5·5	·3015	·4793
2·1	·5137	$\bar{1}$·7107	6·0	·2854	·4554
2·2	·5025	·7010	6·5	·2711	·4332
2·3	·4918	·6918	7·0	·2584	·4123
2·4	·4816	·6825	7·5	·2469	·3925
2·5	·4719	·6738	8·0	·2366	·3740
2·6	·4626	·6652	8·5	·2272	·3564
2·7	·4537	·6568	9·0	·2185	·3395
2·8	·4452	·6486	9·5	·2106	·3234
2·9	·4370	·6405	10·0	·2033	$\bar{1}$·3081
3·0	·4292	$\bar{1}$·6327			

TABLE 2.

INDUCTANCE OF A COIL WOUND WITH 10 TURNS PER CENTIMETRE.

Diameter in Centimetres.

Length in cms.	4	5	6	7	8	9
1	5·78	7·89	10·14	12·49	14·94	17·47
2	16·59	23·28	30·5	38·1	46·2	54·5
3	29·5	42·25	56·02	70·87	86·57	97·15
4	43·4	63·0	84·57	107·9	132·8	159·0
5	58·0	84·92	115·0	147·6	179·6	220·3
6	72·9	107·6	146·7	190·2	236·2	300·2
7	87·9	130·7	179·3	233·1	291·2	353·7
8	103·3	154·2	212·5	277·2	348·0	423·7
9	118·7	177·8	246·1	322·7	406·0	470·5
10	134·2	201·8	280·0	368·0	464·5	568·5
12	165·2	249·3	348·7	460·5	586·0	717·5
14	196·4	298·2	418·2	554·0	702·5	868·7
16	227·7	347·2	487·8	648·2	825·2	1023
18	259·0	395·7	557·8	743·0	950·0	1178
20	290·5	444·7	628·0	838·0	1074	1333
22	321·7	493·7	698·5	933·5	1205	1490
24	353·5	543·0	751·2	1038·7	1322	1647
26	384·0	591·7	889·2	1125	1447	1845
28	416·5	641·2	910·5	1221	1572	1961
30	448·0	690·2	980·5	1316	1697	2120
32	480·0	739·5	1052	1413	1822	2278
34	511·0	788·7	1122	1509	1948	2380

Diameter in Centimetres.

Length in cms.	10	12	14	16	18
1	20·06	—	—	—	—
2	63·1	81·12	99·7	119·5	139·7
3	120·0	155·8	196·0	233·0	273·7
4	186·3	243·5	305·2	369·5	436·0
5	259·2	342·2	430·7	502·3	620·5
6	337·5	448·2	567·5	698·7	823·7
7	420·2	560·7	711·7	873·2	1046
8	504·0	676·7	863·5	1062	1278
9	590·5	800·7	1020	1256	1512
10	679·5	920·2	1181	1464	1762
12	860·7	1175	1518	1886	2283
14	1046	1435	1862	2329	2822
16	1234	1701	2219	2785	3400
18	1424	1970	2577	3245	3962
20	1615	2241	2945	3715	4547
22	1807	2515	3312	4187	5140
24	2000	2790	3682	4667	5735
26	2194	3067	4057	5152	6342
28	2421	3347	4432	5637	6952
30	2582	3625	4807	6125	7565
32	2778	3905	5187	6615	8182
34	2982	4187	5565	7162	8800

TABLE 3.

(Single-Layer Coils.)

S.W.G.	Diameter in millimeters.	Single silk covered.			Double silk covered.		
		Turns per centimeter.	Relative inductance.	Direct-current resistances.	Turns per centimeter.	Relative inductance.	Direct-current resistance.
1	2	3	4	5	6	7	8
40	·122	62·2	9·67	28·4	55·9	7·81	25·5
39	·132	58·8	8·64	22·9	52·8	6·97	20·5
38	·152	52·3	6·84	15·3	47·6	5·66	13·9
37	·173	47·5	5·63	10·8	43·5	4·74	9·90
36	·193	43·3	4·69	7·89	40·0	4·00	7·29
35	·213	40·0	4·00	5·96	36·1	3·26	5·38
34	·234	36·8	3·39	4·57	33·6	2·82	4·18
33	·254	33·2	2·75	3·49	31·5	2·48	3·31
32	·274	32·0	2·56	2·91	29·6	2·19	2·67
31	·295	30·0	2·24	2·35	27·9	1·94	2·18
30	·315	28·2	1·99	1·93	25·5	1·63	1·74
29	·345	25·8	1·66	1·47	23·7	1·40	1·35
28	·376	23·9	1·43	1·15	22·1	1·22	1·06
27	·417	21·8	1·19	·853	20·3	1·03	·794
26	·457	20·0	1·00	·649	18·7	·874	·607
25	·508	18·2	·828	·478	17·1	·731	·449
24	·559	16·7	·697	·363	15·7	·616	·341
23	·610	15·2	·578	·277	14·3	·511	·261
22	·711	13·2	·436	·177	12·8	·410	·171
21	·813	11·6	·337	·115	11·1	·308	·114
20	·914	10·35	·268	·0840	9·9	·245	·0803
19	1·016	9·37	·219	·0616	9·05	·205	·0595
18	1·219	7·87	·155	·0359	7·64	·146	·0349
17	1·422	6·87	·118	·0231	6·50	·106	·0218
16	1·626	5·87	·086	·0151	6·75	·083	·0148
15	1·829	5·23	·068	·0106	5·16	·066	·0105
14	2·032	4·76	·057	·00782	4·65	·054	·0076
13	2·337	4·13	·043	·00513	4·09	·040	·00508
12	2·642	3·68	·034	·0036	3·66	·033	·0036

Columns 4 and 7 give the ratio of inductance of a coil wound with the given wire to that of a similar coil wound with 20 turns per centimetre. (Table 2.)

The resistances in columns 5 and 8 are for a length of 1 centimetre winding on a coil of 10 cms. diameter.

Enamelled wire has approximately the same dimensions as single silk-covered in the smaller gauges.

TABLE 4.

TABLE GIVING THE MUTUAL INDUCTANCE OF TWO CIRCLES FOR VARIOUS RATIOS OF $\frac{r_2}{r_1}$. (See p. 30.)

$\frac{r_2}{r_1}$	M_0	$\frac{r_2}{r_1}$	M_0	$\frac{r_2}{r_1}$	M_0	$\frac{r_2}{r_1}$	M_0
1·00	·000	·51	3·804	·184	14·24	·086	23·32
·99	·010	·50	3·970	·182	14·37	·084	23·61
·98	·022	·49	4·138	·180	14·49	·082	23·90
·97	·038	·48	4·314	·178	14·62	·080	24·20
·96	·059	·47	4·503	·176	14·75	·078	24·50
·95	·082	·46	4·690	·174	14·89	·076	24·82
·94	·107	·45	4·890	·172	15·02	·074	25·16
·93	·137	·44	5·090	·170	15·15	·072	25·50
·92	·170	·43	5·298	·168	15·29	·070	25·84
·91	·204	·42	5·512	·166	15·43	·069	26·02
·90	·240	·41	5·739	·164	15·57	·068	26·20
·89	·278	·40	5·973	·162	15·72	·067	26·39
·88	·320	·39	6·213	·160	15·86	·066	26·56
·87	·365	·38	6·460	·158	16·00	·065	26·75
·86	·410	·37	6·717	·156	16·15	·064	26·95
·85	·456	·36	6·988	·154	16·31	·063	27·14
·84	·507	·35	7·266	·152	16·46	·062	27·34
·83	·563	·34	7·557	·150	16·61	·061	27·54
·82	·616	·33	7·858	·148	16·78	·060	27·73
·81	·675	·32	8·181	·146	16·94	·059	27·95
·80	·735	·31	8·504	·144	17·10	·058	28·16
·79	·797	·300	8·845	·142	17·28	·057	28·38
·78	·865	·295	9·019	·140	17·44	·056	28·60
·77	·933	·290	9·200	·138	17·61	·055	28·83
·76	1·002	·285	9·39	·136	17·78	·054	29·06
·75	1·075	·280	9·58	·134	17·96	·053	29·29
·74	1·154	·275	9·77	·132	18·12	·052	29·52
·73	1·230	·270	9·96	·130	18·31	·051	29·79
·72	1·310	·265	10·17	·128	18·50	·050	30·02
·71	1·394	·260	10·37	·126	18·69	·049	30·27
·70	1·480	·255	10·58	·124	18·88	·048	30·53
·69	1·570	·250	10·79	·122	19·08	·047	30·79
·68	1·665	·245	11·02	·120	19·28	·046	31·04
·67	1·760	·240	11·25	·118	19·48	·045	31·32
·66	1·860	·235	11·47	·116	19·69	·044	31·59
·65	1·963	·230	11·71	·114	19·90	·043	31·90
·64	2·066	·225	11·96	·112	20·11	·042	32·20
·63	2·178	·220	12·21	·110	20·33	·041	32·49
·62	2·293	·215	12·47	·108	20·55	·040	32·78
·61	2·407	·210	12·74	·106	20·77	·039	33·10
·60	2·527	·205	13·01	·104	20·99	·038	33·42
·59	2·650	·200	13·28	·102	21·23	·037	33·76
·58	2·769	·198	13·40	·100	21·48	·036	34·10
·57	2·915	·196	13·52	·098	21·72	·035	34·45
·56	3·052	·194	13·63	·096	21·98	·034	34·78
·55	3·190	·192	13·75	·094	22·24	·033	35·19
·54	3·335	·190	13·87	·092	22·50	·032	35·59
·53	3·486	·188	13·99	·090	22·77	·031	35·97
·52	3·645	·186	14·12	·088	23·04	·030	36·34

TABLE 5.*

POTENTIAL OF A SINGLE WIRE CHARGED WITH ONE UNIT PER CENTIMETRE (NEGLECTING INFLUENCE OF THE EARTH).

$$V = 2\left(\log_e \frac{l}{r} - \cdot 307\right)$$

$\frac{l}{r} =$	5	10	15	20	25	30	35 × 10^3
V_{av}	16·4	17·8	18·6	19·2	19·6	20·0	20·3

$\frac{l}{r}$	40	45	50	60	70	80	90	100 × 10^3
V_{av}	20·6	20·8	21·0	21·4	21·7	22·0	22·2	22·4

TABLE 6.

CAPACITY OF PARALLEL WIRE ANTENNÆ (NEGLECTING INFLUENCE OF THE EARTH): IN MICROMICROFARADS PER METRE.

No. of wires	$\frac{d}{r}$	$\frac{l}{d} = 20$	50	100	150	300
2	100	11·14	9·41	8·35	7·84	7·24
	250	10·20	8·73	7·88	7·46	6·82
	500	9·60	8·29	7·51	7·12	6·55
	1000	9·05	7·88	7·19	6·82	6·26
3	100	13·60	11·15	9·78	9·15	8·20
	250	12·69	10·49	9·29	8·71	7·84
	500	12·07	10·06	8·94	8·40	7·61
	1000	11·48	9·66	8·63	8·14	7·39
4	100	15·58	12·50	10·82	10·03	8·92
	250	14·60	11·88	10·35	9·64	8·60
	500	13·94	11·45	10·03	9·36	8·40
	1000	13·45	11·07	9·71	9·09	8·16
5	100	17·28	13·61	12·00	10·77	9·50
	250	16·28	13·06	11·23	10·39	9·18
	500	15·60	12·60	10·90	10·12	8·99
	1000	15·09	12·22	10·62	9·82	8·79
7	100	20·2	15·37	13·05	11·90	10·40
	250	19·28	14·81	12·67	11·58	10·13
	500	18·52	14·42	12·37	11·34	9·97
	1000	17·98	14·15	12·10	11·14	9·77
10	100	24·1	17·71	14·70	13·28	11·41
	250	23·1	17·10	14·38	13·00	11·21
	500	22·4	16·80	14·10	12·82	11·04
	1000	21·7	16·47	13·85	12·61	10·92
12	100	26·2	18·9	15·45	14·08	11·97
	250	25·3	18·4	15·13	13·78	11·77
	500	24·6	18·04	14·92	13·61	11·65
	1000	24·0	17·70	14·65	13·42	11·48

* This table and several of the following ones are taken from Prof. Howe's articles referred to in the text.

TABLE 7.

POTENTIAL OF A WIRE DUE TO A PARALLEL CHARGED WIRE OF EQUAL LENGTH AND POTENTIAL OF A HORIZONTAL WIRE DUE TO PROXIMITY TO THE EARTH.

$\frac{l}{d}$	E	$\frac{l}{d}$	E	$\frac{l}{d}$	E
·5	·48	8·0	3·78	30	6·27
1·0	·94	8·5	3·88	40	6·81
1·5	1·32	9·0	4·00	50	7·26
2·0	1·64	10	4·20	75	8·05
2·5	1·94	11	4·36	100	8·62
3·0	2·20	12	4·52	200	9·98
3·5	2·42	13	4·64	350	11·1
4·0	2·62	14	4·80	500	11·81
4·5	2·82	15	4·94	750	12·62
5·0	2·98	16	5·06	1000	13·20
5·5	3·18	17	5·18	1300	13·72
6·0	3·28	18	5·28	2000	14·58
6·5	3·42	19	5·38	4000	15·97
7·0	3·54	20	5·46	8000	17·18
7·5	3·66				

TABLE 8 (*a*).

AVERAGE POTENTIAL OF A WIRE LENGTH l' DUE TO UNIT CHARGE PER CENTIMETRE ON A WIRE AT RIGHT ANGLES LENGTH l.

$\frac{l'}{l}$	V_{av}	$\frac{l'}{l}$	V_{av}
0·2	3·31	1·5	1·422
0·4	2·625	2·0	1·20
0·6	2·23	3·0	0·94
0·67	2·134	4·0	0·77
0·8	1·957	6·0	0·58
1·0	1·754		

TABLE 8 (b).

Curve giving the Decrease of the Average Potential of A′C due to the Charge on AB when A and A′ do not coincide.

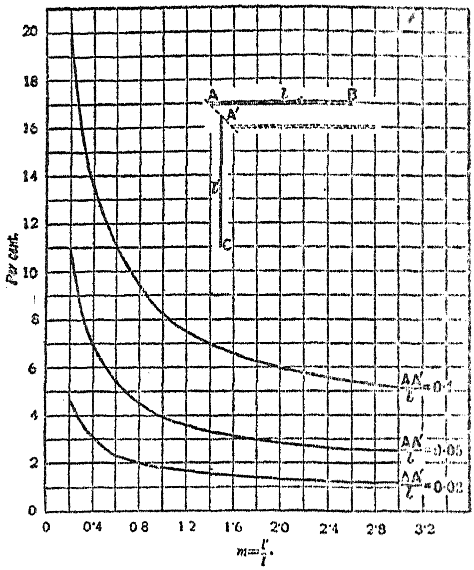


TABLE 9.

Potential on a Wire due to a Uniform Charge of 1 Unit per Centimetre on a Wire of Equal Length inclined at an Angle γ.

γ^0	V_{av}	γ^0	V_{av}	γ^0	V_{av}
2	8·14	20	3·82	80	1·88
4	6·79	25	3·45	90	1·76
6	6·01	30	3·16	100	1·67
8	5·46	35	2·93	110	1·59
10	5·04	40	2·74	120	1·53
12	4·72	45	2·57	140	1·45
14	4·45	50	2·43	160	1·42
16	4·20	60	2·19	180	1·39
18	4·00	70	2·02		

TABLE 10.

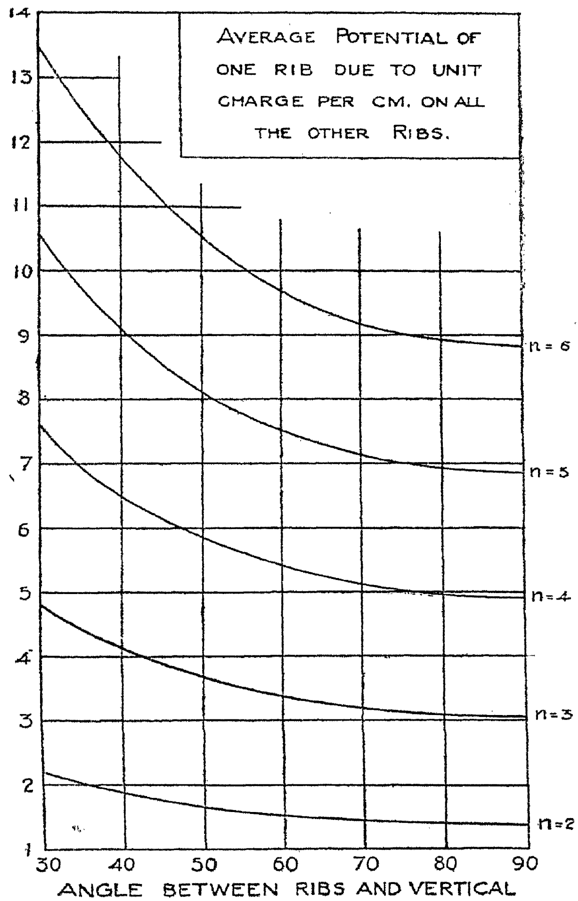
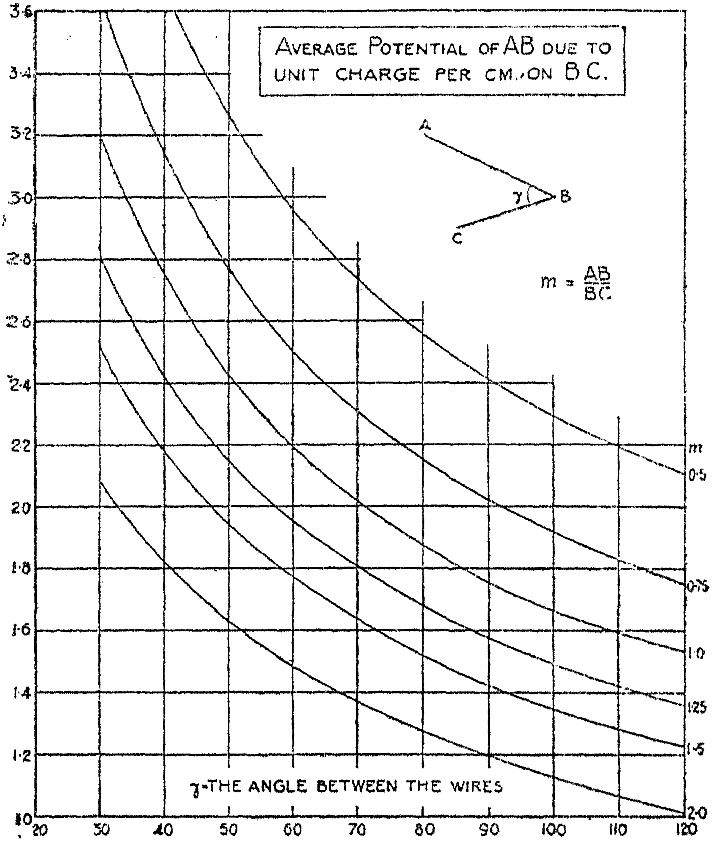

AVERAGE POTENTIAL OF ONE RIB DUE TO UNIT CHARGE PER CM. ON ALL THE OTHER RIBS.
14
13
12
11
10
9
8
7
6
5
4
3
2
1
n = 6
n = 5
n = 4
n = 3
n = 2
30
40
50
60
70
80
90
ANGLE BETWEEN RIBS AND VERTICAL

TABLE 11.



AVERAGE POTENTIAL OF AB DUE TO UNIT CHARGE PER CM. ON BC.
A
B
C
γ
m = AB/BC
m
0·5
0·75
1·0
1·25
1·5
2·0
γ-THE ANGLE BETWEEN THE WIRES
3·6
3·4
3·2
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
20
30
40
50
60
70
80
90
100
110
120

TABLE 12.

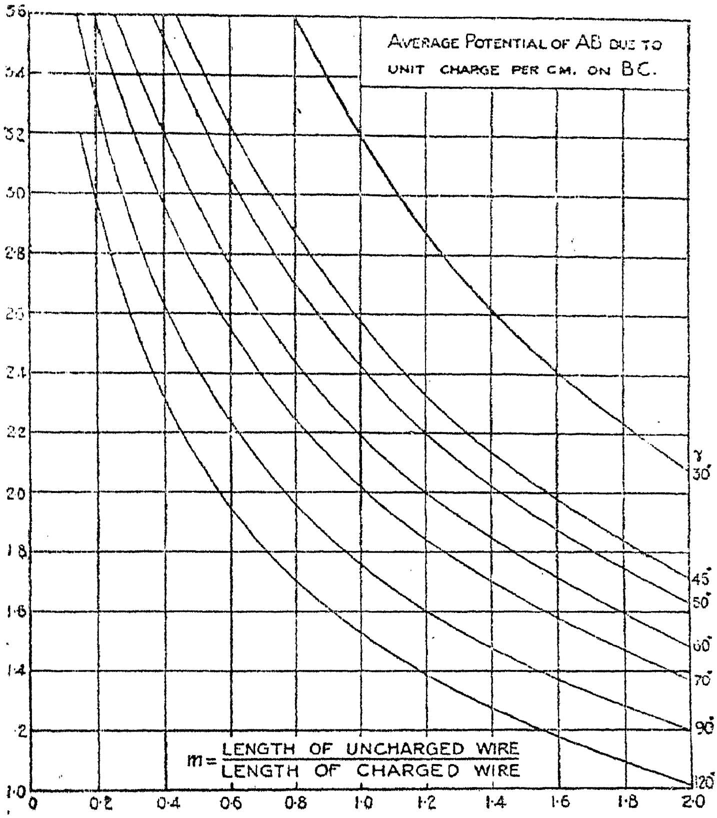

AVERAGE POTENTIAL OF AB DUE TO UNIT CHARGE PER CM. ON BC.
$m = \frac{\text{LENGTH OF UNCHARGED WIRE}}{\text{LENGTH OF CHARGED WIRE}}$
γ
30°
45°
50°
60°
70°
90°
120°
3·6
3·4
3·2
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8
2·0

TABLE 13.

Wave-length of an Aerial with added Series Inductance.

(See page 97.)

$\frac{L_1}{L_0}$	$\frac{\lambda_1}{\lambda_0}$	$\frac{\lambda_0}{\lambda_1}$	Q	$\frac{\lambda_e}{\lambda_m}$
0·00	1·00	1·00	1·57	1·57
0·05	1·04	·962	1·51	1·54
0·10	1·10	·909	1·46	1·50
0·20	1·194	·838	1·314	1·44
0·30	1·29	·777	1·219	1·39
0·40	1·373	·728	1·142	1·35
0·50	1·455	·687	1·078	1·32
0·60	1·530	·651	1·022	1·29
0·70	1·620	·616	·968	1·26
0·80	1·700	·589	·925	1·24
0·90	1·76	·568	·892	1·228
1·00	1·84	·544	·853	1·207
1·50	2·13	·468	·735	1·162
2·00	2·42	·414	·650	1·128
2·50	2·66	·376	·590	1·102
3·00	2·88	·347	·545	1·093
3·50	3·08	·325	·510	1·083
4·00	3·27	·306	·480	1·072
5·00	3·62	·276	·433	1·068
6·00	3·95	·253	·397	1·060
7·00	4·26	·235	·369	1·049
8·00	4·52	·221	·347	1·039
9·00	4·80	·208	·327	1·036
10·0	5·03	·199	·312	1·033
12·0	5·52	·181	·285	1·026
14·0	5·95	·168	·264	1·023
16·0	6·35	·158	·247	1·021
18·0	6·72	·149	·234	1·018
20·0	7·09	·141	·222	1·016
22·0	7·43	·135	·212	1·014
24·0	7·74	·129	·203	1·012

TABLE 14.

WAVE-LENGTH OF AN AERIAL WITH ADDED SERIES CAPACITY.

(See page 98.)

$\frac{C_1}{C_0}$	$\frac{\lambda_1}{\lambda_0}$	$\frac{\lambda_0}{\lambda_1}$
4·0	·917	1·091
3·8	·913	1·095
3·6	·908	1·101
3·4	·904	1·106
3·2	·899	1·112
3·0	·894	1·119
2·8	·888	1·126
2·6	·882	1·134
2·4	·874	1·144
2·2	·866	1·155
2·0	·855	1·170
1·8	·842	1·188
1·6	·830	1·205
1·4	·816	1·225
1·2	·798	1·253
1·0	·774	1·292
·90	·760	1·316
·80	·744	1·344
·70	·727	1·376
·60	·708	1·412
·50	·685	1·460
·45	·675	1·48
·40	·661	1·51
·35	·646	1·55
·30	·630	1·59
·25	·612	1·63
·20	·592	1·69
·15	·571	1·75
·10	·549	1·82
·05	·525	1·91
·00	·500	2·00

INDEX

The following abbreviations have been used :—

G.M.D. for geometrical mean distance.
H.F. „ high frequency.
K „ capacity.
L „ self-inductance,
M „ mutual inductance.

The references are to pages.

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